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# REPRESENTATION OF GENERAL EQUATIONS OF YOUNG'S, SHEAR'S, BULK MODULU'S OF A MATERIAL IN TERMS OF ANISOTROPY CONSTANT A

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**ABSTRACT**:
.In Isotropic material properties are similar in various directions, while anisotropic material properties are different in different directions. In this article, Anisotropy Factor, A is incorporated in the standard equations of Modulus young's, shear, bulk of a material to include anisotropy in the material specimen.

**Keywords**: Young's modulus, Shear modulus, Bulk modulus, Anisotropy Factor

INTRODUCTION:
Youngs modulus is measure of tensile elasticity while Shear modulus − modulus of rigidity is the measure of shear elasticity, Bulk modulus is the measure of volumetric elasticity Standard Equations of the Elastics Modulus of the Material is applicable for properties are assumed to be same in all directions i.e. when the material is isotropic in nature. But in reality the properties are anisotropic i.e. the properties of the material changes with change in crystallographic directions. In order to grep this anisotropic phenomenon, an Anisotropic Factor, A is defined. In this article, The Standard equations are modified to include this Anisotropic Factor, A so that Modified Standard Equations can be used even the properties of the material changes with change in crystallographic directions. When Anisotropic Factor A, equals 1 the material becomes isotropic and Modified Standard Equations which include Anisotropic Factor, A boils down to Standard Equations.
STANDARD EQUATIONS:
$E= 9KG/(3K+G)$; $G=E/2(1+\nu)$;$K=E/3(1-2\nu)$; $\nu = (3K -2G)/(6K+2G)$;$A= 2C_{44}/(C_{11}– C_{12})$; $C_{11} = (S_{11} + S_{12})/ (S_{11} – S_{12}) (S_{11} +2S_{12})$; $C_{12} = (-S_{12})/ (S_{11} – S_{12}) (S_{11} +2S_{12})$;

Modification of Modulus of Elasticities to include anisotropic factor A.
$A= 2C_{44}/(C_{11}– C_{12})$ ; IN TERMS OF COMPLAINCE CONSTANTS
$C_{11} = (S_{11} + S_{12})/ (S_{11} – S_{12}) (S_{11} +2S_{12})$
$C_{12} = (-S_{12})/ (S_{11} – S_{12}) (S_{11} +2S_{12})$
$C_{11} - C_{12} = (S_{11} + S_{12})/ (S_{11} – S_{12}) (S_{11} +2S_{12}) - (-S_{12})/ (S_{11} – S_{12}) (S_{11} +2S_{12})$
$C_{11} - C_{12} = (S_{11} + S_{12}+ S_{12} ) /(S_{11} – S_{12}) (S_{11} +2S_{12}) = (S_{11} + 2S_{12})/ (S_{11} – S_{12}) (S_{11} +2S_{12})$
$C_{11} - C_{12} = 1/(S_{11} – S_{12})$; $C_{44} = 1/ S_{44}= G$ ; $S_{11} = 1/E$ ; $S_{12} = -\nu/E$
$A = 2(S_{11} – S_{12})/ S_{44} = 2G(1/E + \nu/E)$

- ⇨ **$G = EA/2(1+\nu)$ ……..[ I ] If $A=1$, $G=E/2(1+\nu)$;**
- ⇨ $\nu = (3K -2G)/(6K+2G)$
- ⇨ **$1+\nu = 9K/(6K +2G)$**
- ⇨ **$EA/2G = 9K/(6K +2G)$ FROM …[ I ]**
- ⇨ **$EA/G = 9K/(3K +G)$**
- ⇨ **=> $E = 9KG/A(3K +G)$ …..[II ] If $A =1$; $E = 9KG/(3K +G)$**

=> Equating [ I ] and [ II ], we have
=> **$2G (1+\nu)/A = 9KG/A (3K +G)$ Since $E = 2G (1+\nu)/A$ From [I ]**
=> **$6K (1+\nu) + 2G (1+\nu) = 9K$**
=> **$6K (1+\nu) + EA = 9K$**
=> **$EA = 9K -6K (1+\nu)$**
=> **$EA = 3K (3-2(1+\nu))$**
=> **$EA = 3K (1-2\nu)$**
=> **$K = EA/ (1-2\nu)$ …. [III]**

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**III Discussion:**

It is observed that G=E/2(1+ν), in place of E it is found that EA in modified Anisotropic Equation G=EA/2(1+ν), E= 9KG/ (3K+G) in place (3K+G) it is found A(3K+G) in Modified Anisotropic **Equation**

E= 9KG/A(3K+G); K=E/3(1-2ν) in place of E,it is found that EA, in Modified Anisotropic Equation EA/ 3(1-2ν).

In order to check the validity of above equations, data from Ref[3] is taken

G = EA/2(1+ν) ……..[ I ]

E = 9KG/A (3K +G) …..[II]

K = EA/ 3(1-2ν) & K =2G (1+ν)/3(1-2ν) …. [III]

A = $2C_{44} / (C_{11} - C_{12})$ ….. [IV]

ν = (3K -2G)/ (6K+2G) ……. [V ]

| S.No | Material | a(lattice constant) | $C_{11}$ | $C_{12}$ | $C_{44}$(Shear Modulus) | B(Bulk Modulus) | Anisotropy Constant, A | ν |
|---|---|---|---|---|---|---|---|---|
| 1. | $UH_2$ | 5.135 | 99 | 63 | 45 | 75(75 from [I & III]) | 2.5 (from [IV]) | 0.25 (from [V]) |
| 2. | $\alpha - UH_3$ | 3.999 | 221 | 63 | 53 | 116(≈115.6 from [I & III]) | 0.67088 (from [IV]) | 0.3011 (from [V]) |
| 3. | $\beta - UH_3$ | 6.580 | 227 | 102 | 60 | 144(≈143.6 from [I & III]) | 0.96 (from [IV]) | 0.3166 (from [V]) |

| S.No | E = 2G (1+ν)/A | E = 9KG/A (3K+G) | E = 3K (1-2ν)/A |
|---|---|---|---|
| 1. | 45 | 45 | 45 |
| 2. | 205.575 | 205.583 | 205.6359 |
| 3. | 164.575 | 164.578 | 164.6015 |

**In order to check the validity of above equations, data from Ref[4] [ Elastic Stiffness Constants and Mechanical Properties of Scheelite $LiYF_4$] is taken**

| S.No | Material | $C_{11}$ | $C_{12}$ | $C_{44}$ | G(Shear Modulus) | B(Bulk Modulus) | Anisotropy Constant, A | ν |
|---|---|---|---|---|---|---|---|---|
| 1. | $LiYF_4$ [THEROTICAL] | 116 | 55 | 40 | 33 | 81 | 1.311475 (from [IV]) | 0.32 (from [V]) |
| 2. | $LiYF_4$ [THEROTICAL] | 112 | 51 | 34 | 33 | 81 | 1.114754 (from [IV]) | 0.32 (from [V]) |
| 3. | $LiYF_4$ [EXPERIMENTAL] | 121 | 60.9 | 40.6 | 35 | 81 | 1.351 (from [IV]) | 0.31 (from [V]) |

| S.No | E = 2G (1+ν)/A | E = 9KG/A (3K+G) | E = 3K (1-2ν)/A |
|---|---|---|---|
| 1. | 66.429 [THEROTICAL] | 66.46184 [THEROTICAL] | 66.7035 [THEROTICAL] |

| | | | |
|---|---|---|---|
| 2. | 78.1517 [THEROTICAL] | 78.19038 [THEROTICAL] | 78.4747 [THEROTICAL] |
| 3. | 67.87564[EXPERIMENTAL] | 67.9352 [EXPERIMENTAL] | 68.3493 [EXPERIMENTAL] |

**IV Conclusion:**

The Modified Elastic Moduli equation incorporating Anisotropy Constant A, is found to be consistent with standard results for two different sets of data taken from Ref[3] and Ref[4]

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