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MODEL REDUCTION USING EIGEN SPECTRUM AND MODIFIED CAUER FORM

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ABSTRACT
In order to derive the reduced order models of high-order linear time-invariant systems, a combined method combining the modified Cauer form and Eigen spectrum analysis is devised. This approach preserves the pole centroid and system stiffness of both unique and reduced order systems. If the original high-order system is stable and has comparable properties, the suggested procedure ensures the stability of the reduced model.

Keywords:
Eigen spectrum; Modified Cauer Form; Impulse response energy; Integral square error; Order reduction; Stability

I. Introduction

When obtained using theoretical considerations, many physical systems mathematical descriptions frequently result in massive-order models. More precisely, these modeling procedures provide a higher order state space model in the frequency domain representation and a higher order transfer function model in the time domain or state space representation. For control and other applications, it is frequently acceptable to use lower-order state variable or transfer function models as an equivalent representation of such models. For both kinds of reduction, a large number of scholars presented model order reduction strategies. The literature offers a wide range of techniques for order reduction of linear continuous systems in both the time and frequency domains [1–8]. Furthermore, a number of approaches have also been proposed by fusing the characteristics of two distinct approaches [9–14]. Try each of these ways on a different system; each has advantages and disadvantages.

Even with a variety of approaches, none of them consistently produces the greatest outcomes.

The current effort aims to develop an order reduction technique that obtains the reduced order system zeroes by using the modified Cauer form, while maintaining the accuracy of the pole centroid and system stiffness of the original and reduced order systems to obtain the reduced order system poles.

The paper is organised as follow in section (II) statement of problem, section (III) description of method, section (IV) methd for comparison, section (V) numerical examples and section (VI) conclusions.

II. STATEMENT OF THE PROBLEM

Assume that the high-order system's (HOS) transfer function is 'n'

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{b_0 + b_1 s + b_2 s^2 - - - - + b_n s^{n-1}}{(s+\lambda_1)(s+\lambda_2) - - - - - (s+\lambda_n)} \qquad (1)$$

Where -λ1 <-λ2 < ……………< -λn are poles of HOS and that of low-order system (LOS) of order 'r' is

$$G_r(s) = \frac{\widetilde{N}(s)}{\widetilde{D}(s)} = \frac{\alpha_0 + \alpha_1 s + - - - - - \alpha_{n-1} s^{r-1}}{(s+\lambda'_1)(s+\lambda'_2) - - - - (s+\lambda'_n)} \qquad (2)$$

Where $-\lambda'_1 < -\lambda'_2$----------------------$-\lambda'_r$are poles of HOS and that of the low-order system (LOS) of order 'r' is

III. REDUCTION METHOD

The two stages below comprise the reduction technique for obtaining the rth-order reduced models:

Step 1: First, as illustrated in Fig. 1, adjust the Eigen spectrum zone (ESZ) of the HOS.
The ESZ is formed by the two lines that cut through the nearest (Reλ1) and farthest (Reλp) real poles when cut by two lines that pass through the farthest imaginary pole pairs (±Im(max)). This is the case if poles -λ_i (i = 1,n) are positioned at – (Reλi ± Imλi) (i = 1,p) within the ESZ.
Step 2: Measurement of the HOS stiffness and pole centroid:
The mean of the real parts of the poles is known as the pole centroid, and it may be represented as

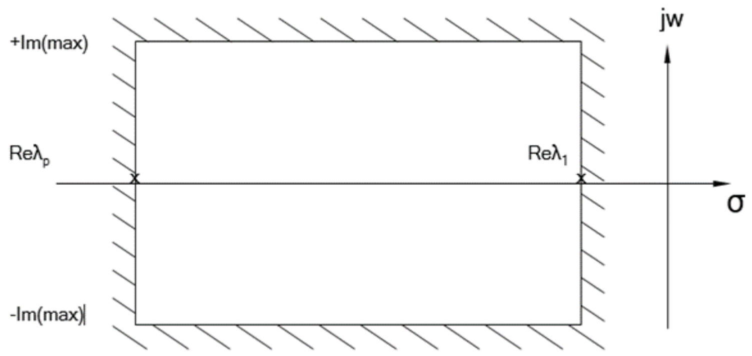
$$\lambda_m = \frac{\sum_{i-1}^{p} R_e\lambda_i}{p} \qquad (3)$$

System stiffness is defined as the ratio of the nearest to the farthest pole of a system in terms of real parts only and is put as

$$\lambda_s = \frac{R_e\lambda_1}{R_e\lambda_p} \qquad (4)$$

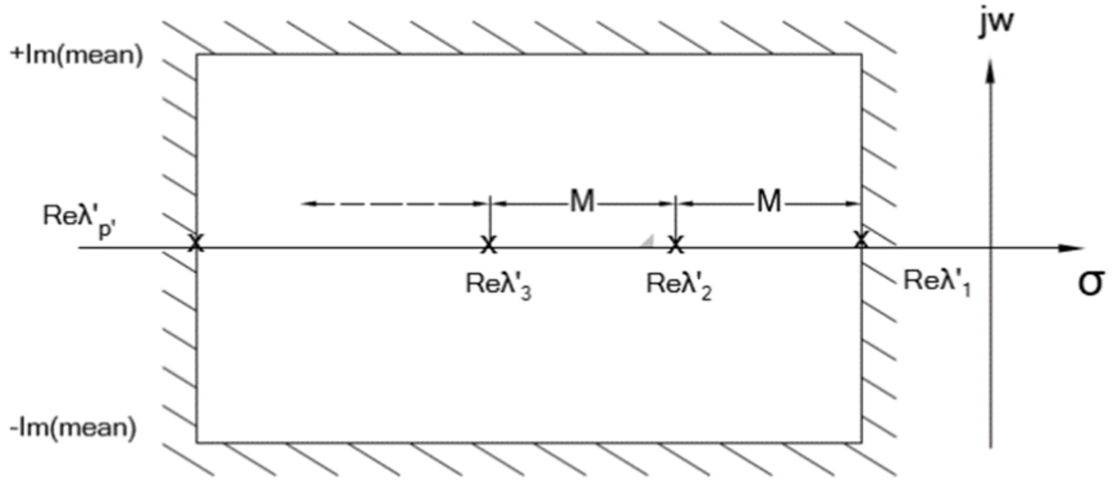
Step 3: Determination of eigen spectral points of LOS:
If λ'_m and $\lambda'_s$are pole centroid and system stiffness of LOS such that $\lambda'_m = \lambda_m$ and $\lambda'_s = \lambda_s$ then following situation arise:

$$\lambda'_s = \frac{R_e\lambda'_1}{R_e\lambda'_{p'}} = \lambda_s$$



(a) eigen spectrum zone (ESZ) of HOS

$$\lambda'_m = \frac{R_e\lambda'_1 + R_e\lambda'_2 + R_e \ldots\ldots + R_e\lambda'_{p'}}{p'} = \lambda_m \qquad (5)$$



(b) eigen spectrum zone (ESZ) of LOS

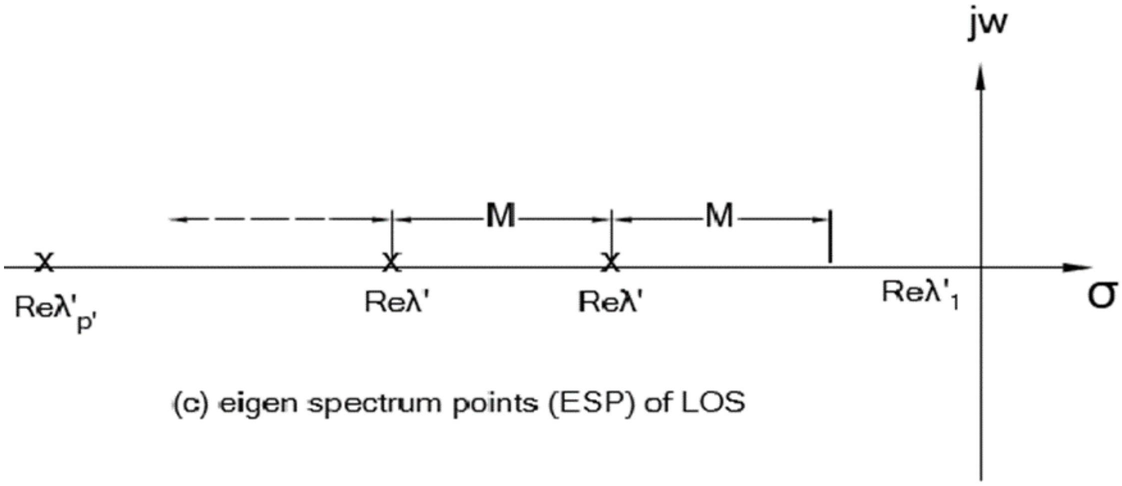


Fig. 1. Eigen spectrum zones and points of system.

Where λ'_i(i=1,r) are the poles of LOS located at – (Reλ'_i ± Imλ'_i) i=1,p'. Now if,

$$\frac{R_e\lambda'_{p'} - R_e\lambda'_1}{p'-1} = M\,, \qquad (6)$$

i.e. $R_e\lambda'_1$+M=$R_e\lambda'_2$, $R_e\lambda'_2$+M=$R_e\lambda'_3$ and so on till $R_e\lambda'_{p'-1} + M = R_e\lambda'_{p'}$ then Eq. (5) can be put as

$$\lambda_m = \frac{R_e\lambda'_1 + R_e\lambda'_{p'} + - - \mp(R_e\lambda'_1 + (p'-2)M)}{p'}$$

Or $\lambda_m p' = R_e\lambda'_1 + R_e\lambda'_{p'} + (M + 2M + \cdots \ldots + (p'-2)M)$

Or N= $R_e\lambda'_1$(p'-1) + $R_e\lambda'_{p'}$+ QM (7)

Where N=λ'_m and QM = M+2M+ ……………………+ (p'-2) M.

By putting$R_e\lambda'_1 = \lambda_s R_e\lambda'_{p'}$. Equation (6) and (7) as under:

$$R_e\lambda'_{p'} - \lambda_s R_e\lambda'_{p'} = M(p'-1) \qquad (8)$$

$\lambda_s R_e\lambda'_{p'}(p'-1) + R_e\lambda'_{p'}$+QM (9)

Eqs. (8) and (9) can be put as

$$R_e\lambda'_{p'}(1-\lambda_s) + M(1-p') = 0$$

$$R_e\lambda'_{p'}[\lambda_s(p'-1)+1] + MQ = N \qquad or$$

$$\begin{bmatrix} \lambda_s(p'-1)+1 & Q \\ (1-\lambda_s) & (1-p') \end{bmatrix} \begin{bmatrix} R_e\lambda'_{p'} \\ M \end{bmatrix} = \begin{bmatrix} N \\ 0 \end{bmatrix} \qquad (10)$$

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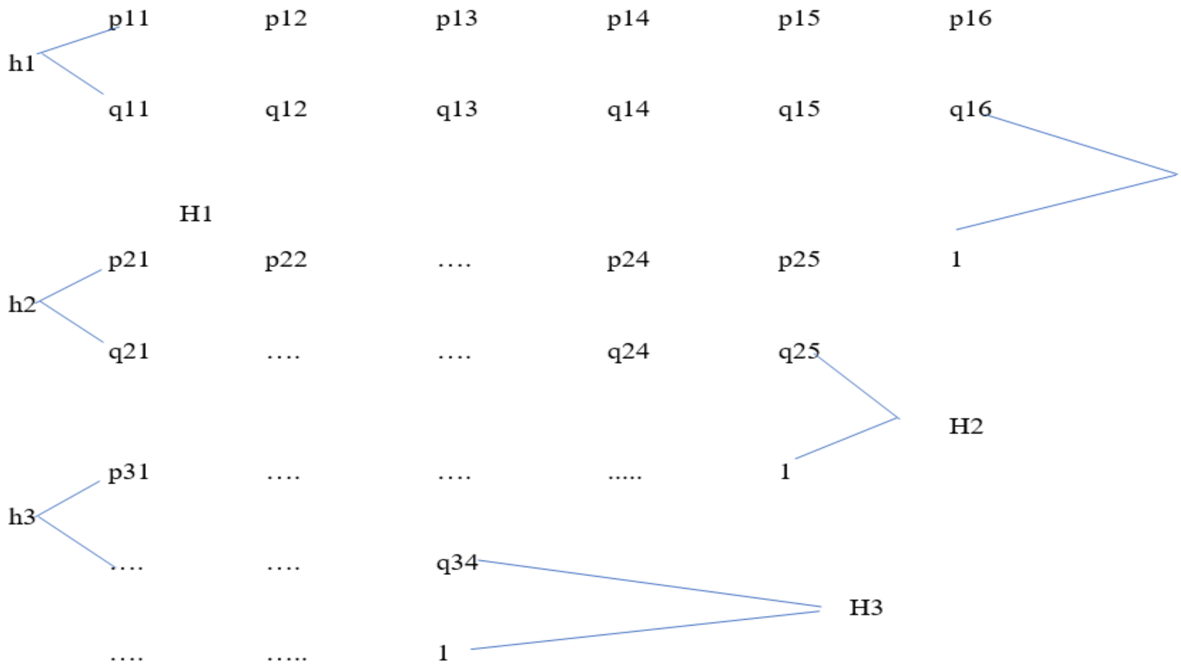
Eq. (10) can be solved for $R_e\lambda'_{p'}$ and M enabling thereby to locate the Eigen spectral points (ESP) as shown in Fig. 1.
By applying the algorithm [10], the first ‘r’ quotients of the modified Cauer form of a continued fraction, viz. h1, H1, h2, H2 are evaluated.
Now a modified Routh array for r = 6 is built as given below:

Step-2
By applying the algorithm [10], the first ‘r’ quotients of the modified Cauer form of a continued fraction, viz. h1, H1, h2, H2 are evaluated.
Now a modified Routh array for r = 6 is built as given below:



Where the first two rows are formed from the denominator and numerator coefficients of Gr(s) in (2) and the remaining entries in the array are obtained by the algorithm given in [10-12].

$$Nk(s)=\sum_{i=0}^{k-1} \propto_i s^i \qquad (7)$$

IV. METHOD FOR COMPARISON

Using Matlab/Simulink, the relative integral square error (ISE) index between the transient portions of the reduced models and the original system is computed in order to assess the accuracy of the suggested method. The definition of the relative integral square error, or RISE, is defined as

$$ISE=\int_0^\infty [y(t) - y_k(t)]^2 \, dt$$

V. NUMERICAL EXAMPLES

The proposed method explains by considering numerical example, taken from the literature. The goodness of the proposed method is measured by calculating integral square error (ISE) between the transient parts of the original and reduced model using MATLAB. The ISE should be minimum for better approximation i.e close the Rk(s) to Gn(s) , which is given by

$$ISE=\int_0^\infty |y(t) - y_k(t)|^2 dt$$

Where, y(t) and yk(t) are the unit step responses of original and reduced system respectively.

Example:-

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Let us consider a tenth-order system that was previously studied by Edgar [15] and Mukherjee [11]. The system has very high gain and no numerator dynamics. All of the system's poles are real.
The HOS transfer function G4(s) is given as

$$G_4(s) = \frac{24 + 24s + 7s^2 + s^3}{24 + 50s + 35s^2 + 10s^3 + s^s}$$

Where $\lambda_1 = -1, \lambda_2 = -2, \quad \lambda_3 = -3, \quad \lambda_4 = -4$
The impulse response energy (IRE) of the aforementioned system is 0.5.
For both the original and other lower order models, the impulse response energy (IRE) is computed and is provided by
IRE = $\int_0^\infty g^2(t)dt$
Where the system's impulse response is represented by g (t).
In the event that a second-order model G2(s) is needed, the G2(s) can be obtained as follows by following the instructions in Section 2.

$$G_2(s) = \frac{s + 4}{s^2 + 5s + 4}$$

With 3.57×10-3, 0.5 for the ISE and IRE, respectively.
Figure 3 displays the step response for both the original and reduced order models. Table 2 contrasts the suggested approach with the various approaches currently in use for a second-order reduced model.
If a second-order model $G_2(s) = \frac{\tilde{N}(s)}{\tilde{D}(s)}$ is to be synthesized using this method, steps to be followed are as under:
Step 1: Correcting the HOS's ESZ:
It will be a line connecting the closest and furthest poles because all poles are real.
Step 2: Measurement of the HOS stiffness and pole centroid:

$$\lambda_m = \frac{\sum_{i=1}^{10} \lambda_i}{10} = 2.5$$

$$\lambda_s = \frac{\lambda_1}{\lambda_4} = 0.25$$

Step 3: Determination of Eigen spectral points of LOS:
Eq. (10) can be formed as under:

$$\begin{bmatrix} 1.25 & 0 \\ 0.75 & -1 \end{bmatrix} \begin{bmatrix} \lambda'_{2'} \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$\lambda'_{2'} = 4$$

$$\lambda'_{1'} = 1$$

Therefore $\quad \widetilde{D}_2(s) = s^2 + 5s + 4$

Step 4: Following Eq. (12), we have

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$$
\begin{array}{lccccc}
 & 24 & 50 & 35 & 10 & 1 \\
h_1 = 1 & & & & & \\
 & 24 & 24 & 7 & 1 & \\
 & & & & & = H_1 \\
 & 26 & 28 & 9 & 1 & \\
h_2 = -13 & & & & & \\
 & -2 & -4 & -2 & &
\end{array}
$$

Construct the modified Routh array as described in reduction method:

$$
\begin{array}{lccc}
 & 4 & 5 & 1 \\
h_1 & & & \\
 & 4 & 1 & \\
 & & & H_1 \\
 & & 1 &
\end{array}
$$

Thus Reduced Numerator is given as

$$\tilde{N}_2(s) = s + 4$$

$$G_2(s) = \frac{s+4}{s^2 + 5s + 4}$$

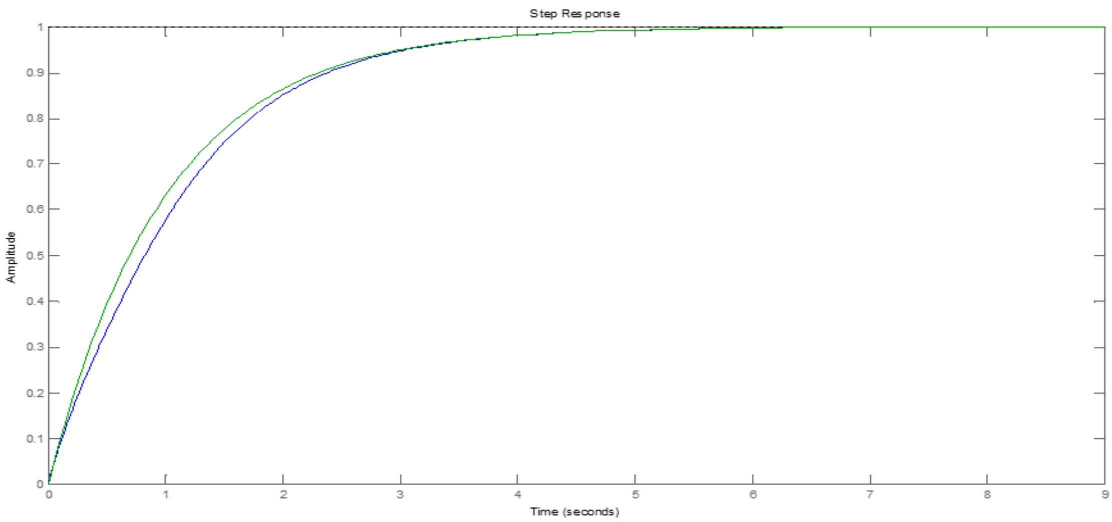

Figure1. Step Response Comparison between original system and reduced system

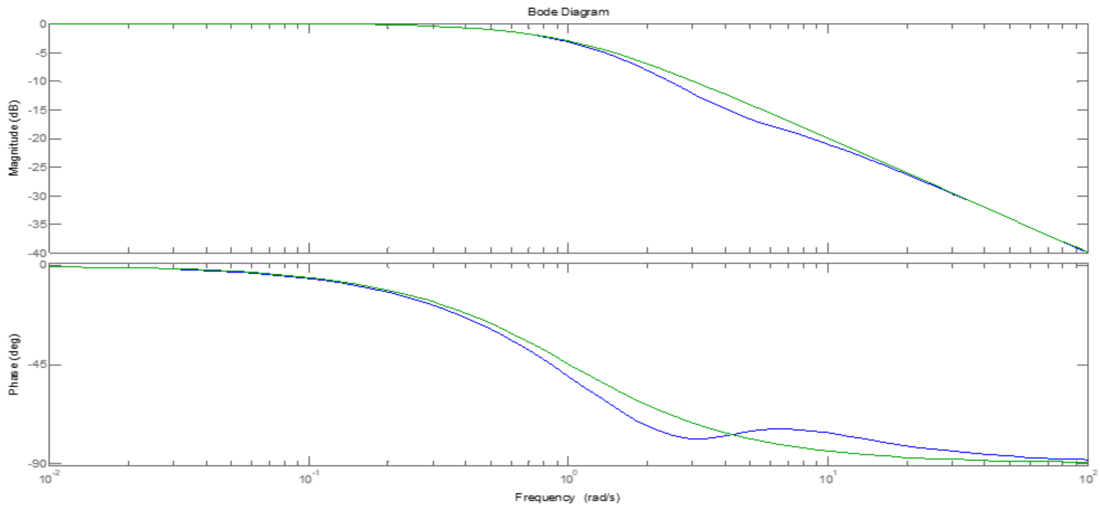

Figure 2 Bode Plots of original system and reduced system

COMPARISON OF THE REDUCED SYSTEM WITH ORIGINAL SYSTEM

Table-1

Parameters	Original System	Reduced System
Rise Time	0.8000	0.8000
Settling Time	1.9800	1.9800
Peak	71	71
PeakTime	1	1
SettlingMin	49	50
SettlingMax	49	50
Overshoot	44.8980	42.0000
Undershoot	0	0

Table-2

Comparison of reduced order models obtained through proposed and other methods for Example.

Method of order reduction	Reduced models	ISE	IRE
Proposed method	$G_2(s) = \frac{s+4}{s^2+5s+4}$	3.57×10-3	0.5
Krishnamurthy and Seshadri [16]	$G_2(s) = \frac{20.1457s+24}{30s^2+42s+24}$	9.5891×10-3	4.69766×10^{-1}
Gutman et al.[17]	$G_2(s) = \frac{96s+288}{70s^2+300s+288}$	4.5593×10-2	7.54249×10^{-1}
Moore [18]	$G_2(s) = \frac{0.8217s+0.4543}{s^2+1.268s+0.4663}$	1.56955×10-4	4.62532×10^{-1}
Safonov and Chiang [19]	$G_2(s) = \frac{s+5.403}{s^2+8.431s+4.513}$	4.51613×10-2	4.66063×10^{-1}

VI. CONCLUSIONS

An order reduction technique was put out by the authors for the high order linear single-input single-output systems. The Eigen Spectrum approach is utilized to determine the denominator polynomial of the simplified model, and the Modified Caur Form method is employed to compute the numerator coefficients. The suggested method has the advantages of stability, simplicity, efficiency, and computer orientation. An example from the literature has been used to explain the suggested algorithm. Figures 1 and 2, respectively, display the step responses and Bode plots of the original and simplified system of second order.

We can conclude that the suggested technique is comparable in quality by comparing the time domain specification of the reduced order system and higher order system approach with the other well-known order reduction methods in the literature, as shown in Table-I.

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