Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 7, No.3, July : 2024

FREE CONVECTIVE HEAT AND MASS TRANSFER THROUGH A POROUS MEDIUM WITH PERIODIC PERMEABIIITY IN A VERTICAL CHANNEL WITH DISSIPATIVE EFFECTS

Dr. M.Sreevani, Department of Mathematics, College of Engineering & Technology, SK University, Ananthapuramu-515 003, AP., India

ABSTRACT: In this paper we study the combined free convective heat and mass transfer of a viscous incompressible fluid through a highly porous medium confined in a vertical channel bounded by flat plates. Using perturbation method, the velocity and temperature have been obtained and analysed computationally for various parameters. The rate of heat transfer has been evaluated.
Key words: Periodic permeability, Dissipative effects.

1. INTRODUCTION:

Combined heat and mass transfer occurs due to the existence of temperature and concentration variations in nature and in many Industrial applications force. For example, in controlling surface temperature by evaporation cooling, controlling polymerization reaction Products by injecting suitable reactants along the porous wall of the reactor, distillation of volatile components from a mixture with non-volatiles, are a few technological processes in which mass transfer accompanies the transfer of heat. The understanding of this transport processes is desirable in order to effectively control the overall transport characteristics .The combined effects of thermal and mass diffusion in channel flows has been studied in the recent times by a few authors, notably, Nelson and Wood(4),Lee et al(2), Miyatake and Fujii (3), Sparrow et al(9) and others(10-12). Nelson and Wood (4), Trevison and Bejan (10) have analysed natural convection heat and mass transfer through a vertical porous layer subjected to uniform flumes of heat and mass from the side. Yan and Lin (13) have examined the effects of the latent heat transfer associated with the liquid film vaporization on the heat transfer in the laminar forced convection channel flows.

In all the studies cited above the permeability of the porous medium has been assumed as constant. In fact, a porous material containing the fluid is a non-homogeneous medium and there can be numerous in-homogeneities present in a porous medium. Therefore the permeability of the porous medium may not necessarily be constant. Several authors (1,6,7,8) have analysed the convective flows with periodic porous permeability under different conditions.

In this paper we consider the combined free convective heat and mass transfer flow of a viscous incompressible fluid through a highly porous medium confined in a vertical channel bounded by plat plates. Assuming that the flow takes place at low concentration we neglect the Soret and Doufer effect .The viscous and Darcy dissipations are taken into account .The non-linear equation governing the flow, heat and mass transfer have been solved by using a perturbation technique .The velocity, temperature and concentration have been analysed for variations in the governing parameters .The shear stress, the rate of heat transfer has been evaluated for different variation.

2. FORMULATION OF THE PROBLEM:

We consider the flow of a viscous incompressible fluid through a highly porous medium confined in a vertical channel bounded by two flat plates .We choose a rectangular Cartesian coordinate system O(x,y,z) with the plates in the x-y plane .The z-axis is taken normal to the plane of the plates .The walls are maintained at constant temperature T0 & T1 and constant concentrations C0 & C1 in the presence of temperature dependent heat source of strength Q. The permeability of the porous medium is assumed to be of the form

Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 7, No.3, July : 2024

$$K(y) = \frac{K_0}{1+\varepsilon\cos(\pi y/d)} \quad (1)$$

Where K_0 is the mean permeability of a medium, d is the wave length of the permeability distribution and ε(<1) is the amplitude of the permeability variation. Since the fluid extends to infinity in X-direction it follows from the equation of continuity that

$$\frac{\partial u}{\partial x} = 0 \quad \Rightarrow \quad u = u(y,z) \quad (2)$$

All the field properties are assumed constant except that the influence of the density variation with the temperature and concentration is considered only in the body force term. The viscous dissipation and Darcy dissipation are taken into account. Assuming that the flow takes place at low concentration. We neglect Soret and Doufer effect.

The governing equations are

$$\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\mu u}{k(y)}-\rho g-\frac{\partial p}{\partial x}=0 \quad (3)$$

$$\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\mu}{k_1}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right]+\frac{\mu}{K_1(y)}u^2=0 \quad (4)$$

$$\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}=0 \quad (5)$$

The boundary conditions are

u =0 on z = 0 and z = d
T = To, C = C0 on z = 0 (6)
T = T1, C = C1 on z = d

Introducing the following non-dimensional quantities

$$(y^*, z^*) = (y,z)/d,\; u = \left(\frac{\mu}{\rho_0 d}\right)u^*,\; \theta^* = \left(\frac{T-T_0}{T_1-T_0}\right),\; C^* = \left(\frac{C-C_0}{C_1-C_0}\right)$$

The equations (3)-(5) reduce to (dropping the asterisk)

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}-D^{-1}(1+\varepsilon\cos\pi y)u+G(\theta+Nc)=0 \quad (7)$$

$$\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}+Ec.\Pr\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right]+Ec.\Pr D^{-1}u^2=0 \quad (8)$$

$$\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}=0 \quad (9)$$

Where

$G = \dfrac{\beta g(T-T_0)d^3}{\nu^2}$ is the Grashof number $\quad$ $D^{-1} = \dfrac{d^2}{k_0}$ is the Darcy number

$P_r = \dfrac{\mu C_p}{k_1}$ is the Prandtl number $\quad$ $Ec = \dfrac{\nu^2}{d^2(T-T_0)C_p}$ is the Eckert number.

The corresponding boundary conditions are

u = 0 on z = 0 and z = 1
θ = 0, C = 0 on z = 0 (10)

Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 7, No.3, July : 2024

θ = 1, C = 1 on z = 1

3.METHOD OF SOLUTTION:

To Solve the equations (7)- (9) with respect to the boundary conditions (10) using the perturbation technique. Assuming Ec(<<1) to be small, we take the asymptotic expansions of velocity , temperature and concentration as

$$u = u_0 + Ecu_1 + O(Ec^2)$$
$$\theta = \theta_0 + Ec\theta_1 + O(Ec^2)$$
$$C = C_0 + EcC_1 + O(Ec^2) \quad (11)$$

After Solving the equations (7)- (9) by substitution (11) with respect to the boundary conditions (11) , considering the periodic variation of permeability and we take

$$u_0 = u_{00}(z) + \varepsilon\cos(\pi y)u_{01}(z) \qquad \theta_0 = \theta_{00}(z) + \varepsilon\cos(\pi y)\theta_{01}(z)$$

$$C_0 = C_{00}(z) + \varepsilon\cos(\pi y)C_{01}(z) \qquad u_1 = u_{10}(z) + \varepsilon\cos(\pi y)u_{11}(z)$$

$$\theta_1 = \theta_{10}(z) + \varepsilon\cos(\pi y)\theta_{11}(z)$$

$$C_1 = C_{10}(z) + \varepsilon\cos(\pi y)C_{11}(z) \quad (12)$$

Substituting (12) and equating the coefficients of ε we get

$$\frac{\partial^2 u_{00}}{\partial z^2} - D^{-1}u_{00} = -G(\theta_{00} + NC_{00}) \quad (13)$$

$$\frac{\partial^2 \theta_0}{\partial z^2} = 0 \quad (14)$$

$$\frac{\partial^2 C_{00}}{\partial z^2} = 0 \quad (15)$$

$$\frac{\partial^2 u_{10}}{\partial z^2} - D^{-1}u_{10} = -G(\theta_{10} + NC_{10}) \quad (16)$$

$$\frac{\partial^2 \theta_{10}}{\partial z^2} = -P(\frac{\partial u_{00}}{\partial z})^2 - PD^{-1}u_{00}^2 \quad (17)$$

$$\frac{\partial^2 C_{10}}{\partial z^2} = 0 \quad (18)$$

The respective boundary conditions are

$$u_{00} = 0,\ u_{10} = 0 \ on\ z = 0 \ \&\ z = 1$$
$$\theta_{00} = 0\ ,\ \theta_{10} = 0\ ,\ C_{00} = 0, C_{10} = 0\ on\ z = 0$$
$$\theta_{00} = 1,\ \theta_{10} = 0\ , C_{00} = 1,\ C_{10} = 0\ on\ z = 1 \quad (19)$$

and

Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 7, No.3, July : 2024

$$\frac{\partial^2 u_{01}}{\partial z^2} - (\pi^2 + D^{-1})u_{01} = D^{-1}u_{00} - G(\theta_{01} + NC_{01}) \tag{20}$$

$$\frac{\partial^2 \theta_{01}}{\partial z^2} - \pi^2 \theta_{01} = 0 \tag{21}$$

$$\frac{\partial^2 C_{01}}{\partial z^2} - \pi^2 C_{01} = 0 \tag{22}$$

$$\frac{\partial^2 u_{11}}{\partial z^2} - (\pi^2 + D^{-1})u_{11} = D^{-1}u_{10} - G(\theta_{11} + NC_{11}) \tag{23}$$

$$\frac{\partial^2 \theta_{11}}{\partial z^2} - \pi^2 \theta_{11} = -2P(\frac{\partial u_{00}}{\partial z})(\frac{\partial u_{01}}{\partial z}) - 2PD^{-1}u_{00}u_{01} \tag{24}$$

$$\frac{\partial^2 C_{11}}{\partial z^2} - \pi^2 C_{11} = 0 \tag{25}$$

with boundary conditions

$$u_{01} = 0,\ u_{11} = 0 \ on\ z = 0 \,\&\, z = 1$$
$$\theta_{01} = 0,\ \theta_{11} = 0,\ C_{01} = 0, C_{11} = 0 \ on\ z = 0 \,\&\, 1 \tag{26}$$

Solving the differential equations (13)-(18) and (20)-(25) with the respective boundary conditions (19) and (26) we obtain

$$u_{00} = a_1 e^{hz} + a_2 e^{-hz} + a_3 z$$
$$\theta_{00} = z\,, \quad C_{00} = z$$
$$u_{01} = a_7 e^{h_1 z} + a_8 e^{-h_1 z} + a_4 e^{hz} + a_5 e^{-hz} + a_6 z$$
$$\theta_{01} = 0\,,\ C_{01} = 0$$
$$u_{10} = (b_3 + + a_{63} z^2 + a_{65} z + a_{67})e^{hz} + (b_4 + a_{64} z^2 + a_{66} z + a_{68})e^{-hz} +$$
$$+ a_{61} e^{2hz} + a_{62} e^{-2hz} + a_{69} z^4 + a_{70} z^2 + b_1 z + b_2$$
$$\theta_{10} = a_{19} e^{2hz} + a_{20} e^{-2hz} + (a_{21} z + a_{23})e^{hz} + (a_{22} z + a_{24})e^{-hz} +$$
$$+ a_{25} z^4 + a_{26} z^2 + a_{17} z + a_{18}$$
$$C_{10} = 0$$

$$u_{11} = (b_{36} + b_{34} + b_{30} z + b_{32} z^2)e^{h_1 z} + (b_{37} + b_{31} z + b_{33} z^2 + b_{35})e^{-h_1 z} +$$
$$+ (b_{12} + b_{14} z + b_{16} z^2)e^{hz} + (b_{13} + b_{15} z + b_{17} z^2)e^{-hz} + b_{18} e^{2hz} + b_{19} e^{-2hz} +$$
$$+ b_{24} e^{\pi z} + b_{25} e^{-\pi z} + b_{26} e^{(h+h_1)z} + b_{27} e^{(h-h_1)z} + b_{28} e^{(h_1-h)z} + b_{29} e^{-(h+h_1)z} +$$
$$+ b_{20} z^4 + b_{21} z^2 + b_{22} z + b_{23}$$

$$\theta_{11} = a_{59}e^{\pi z} + a_{60}e^{-\pi z} + a_{43}e^{(h+h_1)z} + a_{44}e^{(h-h_1)z} + a_{45}e^{(h_1-h)z} + a_{46}e^{-(h+h_1)z} +$$

$$+ a_{47}e^{2hz} + a_{48}e^{-2hz} + (a_{49} + a_{51}z)e^{hz} + (a_{50} + a_{52}z)e^{-hz} + (a_{53} + a_{55}z)e^{h_1 z} +$$

$$+ (a_{54} + a_{56}z)e^{-h_1 z} + a_{57}z^2 + a_{58}$$

C11=0

Where the constants a1,a2,…..a70,b1,b2,…..b7 constants.

4. SHEAR STRESS AND NUSSELT NUMBER:

The shear stress on the plates are given by

$$\tau = \mu(\frac{\partial u}{\partial z})_{z=0,d}$$

which in the non-dimensional form reduce to

$$\tau\bullet = \frac{\tau}{(\mu^2 / \rho_0 d^2)} = (\frac{\partial u}{\partial z})_{z=0,1}$$

$$= [(u'_{00} + Ecu'_{10}) + \varepsilon\cos(\pi y)(u'_{01} + Ecu'_{11})]_{z=0,1}$$

The expressions for τ at the plates are

$$(\tau)_{z=0} = (b_{42} + Ec\,b_{38}) + \varepsilon\cos(\pi y)(b_{44} + Ec\,b_{40})$$

$$(\tau)_{z=1} = (b_{43} + Ecb_{39}) + \varepsilon\cos(\pi y)(b_{45} + Ecb_{41})$$

From the temperature field the rate of heat transfer coefficient in terms of the Nusselt number (Nu) is given by

$$Nu = -(\frac{q_w d}{k_1(T_1 - T_0)}) = (\frac{\partial\theta}{\partial z})_{z=0,1}$$

$$= [(\theta'_{00} + Ec\theta'_{01}) + \varepsilon\cos(\pi y)(\theta'_{01} + Ec\theta'_{11})_{z=0,1}$$

The corresponding expressions for Nu at the plates are

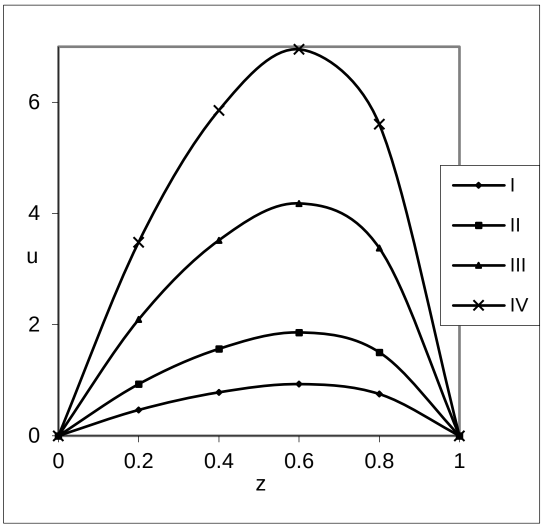
$$(Nu)_{z=0} = 1 + Ec(b_{46} + \varepsilon\cos(\pi y)b_{48})$$

$$(Nu)_{z=1} = 1 + E(b_{47} + \varepsilon\cos(\pi y)b_{49})$$

5. RESULTS & DISCUSSION:

For the physical interpretation of the problem numerical calculations are carried out for different parameters G, D, N & y. Also the value of the Prandtl number P_r is chosen to be 0.71. The velocity (u) and temperature distribution (θ) have been exhibited for different variations in the governing parameters thermal Grashof number G, Darcy number D, buoyancy ratio N & distance y . Fig. (1) Exhibits the effect of velocity fields for fixed y = 0.25, $D^{-1} = 10^3$, N=1for different Groshofs number (G= 10^3, 3×10^3, 5×10^3 , 10^4).The velocity rises from its value zero to attain maximum at z=0.6 and then falls gradually to its prescribed value on the boundary z =1. It is found that the velocity enhances with increase in the thermal buoyancy G with maximum at z = 0.6. Figure 2 represents the effect of velocity profiles for different Darcy Parameter (D^{-1}= 10^3,3×10^3, 5×10^3 ,10^4 ,

2×10^4 with $G = 10^3$, $N = 1$, $So = 0.25$, it is found that a decrease in the permeability of the porous medium reduces the velocity in the entire fluid region.



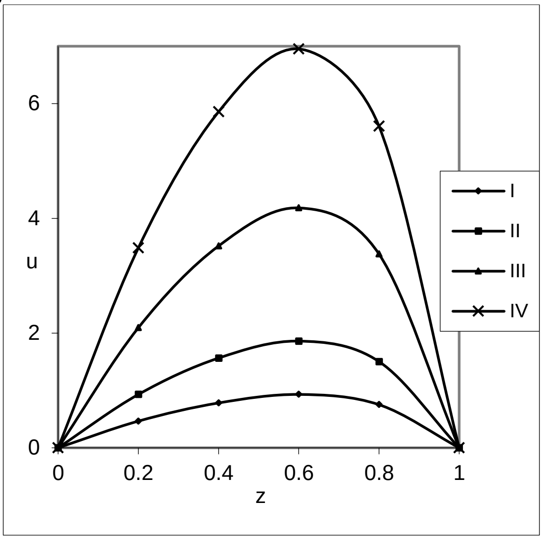
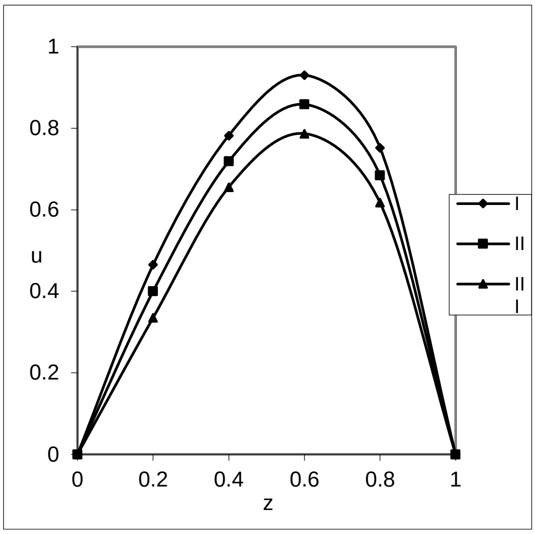


Fig.1 Velocity profile for different G Fig.2 Velocity profile for different D^{-1}

The velocity profiles for different y(y = 0.25, 0.5, 0.75), $G=10^3, D^{-1}=10^3$ are studied and represented in figure 3. It is observed that an increase in y depreciates the velocity u. Figure 4 shows that when the concentration buoyancy force dominates over the thermal buoyancy force the velocity u enhances with N when the two forces act in the same direction and reduces when they act in opposite directions for different N(N =1, 2,-0.5, -0.8).



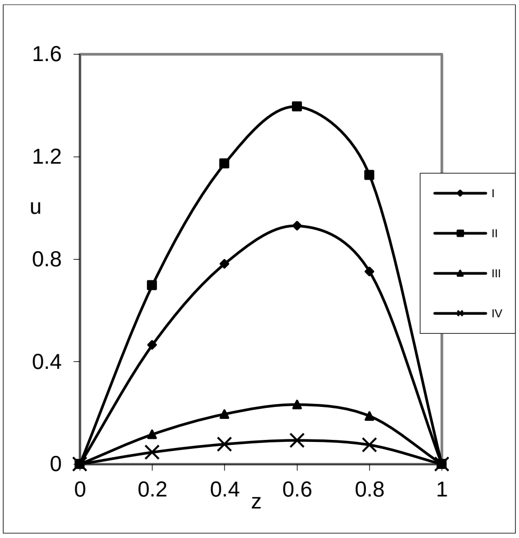


Fig.3: Velocity profiles for different y Fig:4: velocity profiles for different N

Fig.5 shows the variation of the temperature θ for different buoyancy ratio (N=1, 2, -0.5, -0.8)for $G = 10^3$, $D^{-1}=10^3$, y = 0.5 . We found that with the concentration buoyancy force dominating over the thermal buoyancy force the temperature experiences an enhancement when the two buoyancy forces act in the same directions while it reduces when they act in opposing directions. From fig.6, we observe the temperature profiles for different horizontal distance (y= 0.25,0.5,0.75) for fixed $G = 10^3$, $D^{-1}=10^3$, N=1we observe that the temperature is positive for $y \le 0.5$ and is totally negative for y = 0.75.Also θ enhances with increase in horizontal distance y with maximum attained at z = 0.6.

Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 7, No.3, July : 2024

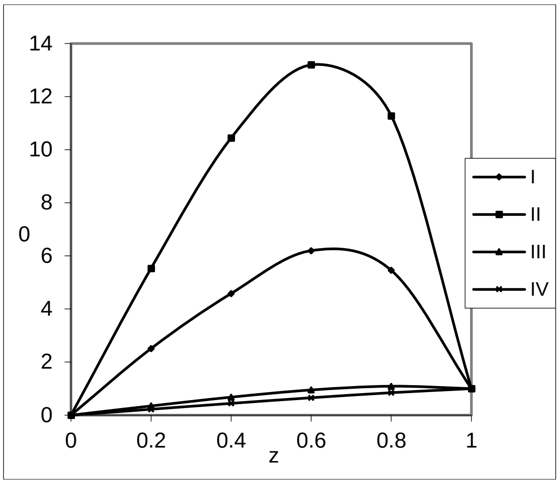


Fig.7 θ with N

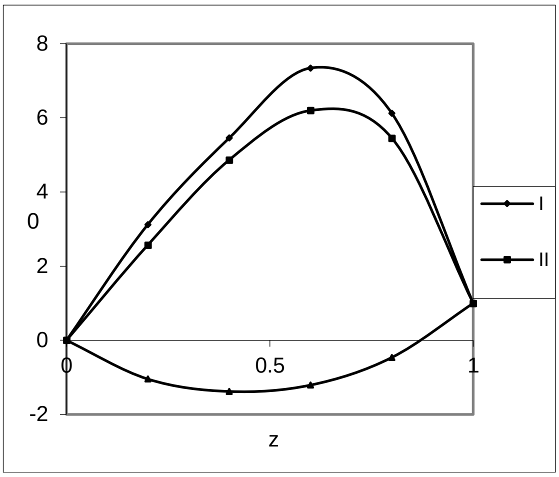


Fig.8 θ with y

The shear stress (τ) at the boundaries have been evaluated in tables 1 & 2 for different variations of the governing parameters G,D^{-1},N & y and the rate of heat transfer (Nusselt number) on the boundaries is exhibited in tables.3&4.

Table.1

Shear Stress (τ) at z = 0

Sc=1.3, P=0.71

D^{-1}	I	II	III	IV	V	VI	VII	VIII
10^3	-2.5599	-3.8046	-2.2539	-1.6821	-0.7284	-0.3244	-1.8673	-1.1421
$3x10^3$	1.9366	2.9733	6.2103	1.7967	0.0058	-0.0406	-0.0746	-2.0847
$5x10^3$	0.0483	1.0062	3.7833	0.0871	-0.0176	-0.0103	0.2371	0.4258
10^4	0.1351	0.8358	2.6679	0.1232	0.0162	0.0052	0.2958	0.4564

Table.2

Shear Stress (τ) at z = 1

Sc=1.3, P=0.71

D^{-1}	I	II	III	IV	V	VI	VII	VIII
10^3	-3.6729	-7.602	-12.588	-5.411	-8.678	-3.392	-16.496	2.634
$3x10^3$	-3.9298	-15.095	-40.731	-6.802	-7.521	-2.794	-11.745	5.793
$5x10^3$	-2.073	-6.0163	-11.199	-3.971	-6.132	-2.409	-9.694	6.334
10^4	-2.418	-5.733	-10.839	-3.741	-5.746	-2.259	-8.456	7.266

Table.3

Nusselt Number(Nu) at z = 0

Sc=1.3, P=0.71

D^{-1}	I	II	III	IV	V	VI	VII	VIII
10^3	0.1612	-2.3839	-6.6258	0.0993	0.9566	1.0011	0.1071	0.0531
$3x10^3$	0.9878	0.9225	0.8137	0.6606	1.0083	1.0094	0.8895	0.7912
$5x10^3$	0.9971	0.9593	0.8964	0.8813	1.0088	1.0096	0.9726	0.9482
10^4	1.0102	1.0119	1.0149	0.9109	1.0096	1.0098	0.9906	0.9709

Table.4

Nusselt Number(Nu) at z = 1

Sc=1.3, P=0.71

D^{-1}	I	II	III	IV	V	VI	VII	VIII
10^3	-7.4406	-6.3763	-22.096	-17.1303	2.1211	2.5832	-5.6378	-5.1456
$3x10^3$	-16.833	-13.467	-36.1279	-38.1133	2.0418	2.6956	-16.775	-16.713
$5x10^3$	-18.541	-15.843	-53.8384	-43.7614	0.3757	1.4351	-18.4456	-18.3512
10^4	-19.451	-16.512	-56.0654	-45.647	0.1959	1.2961	-19.4022	-19.3531

Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 7, No.3, July : 2024

G	$2x10^3$	$3x10^3$	$5x10^3$	$2x10^3$	$2x10^3$	$2x10^3$	$2x10^3$	$2x10^3$
N	1	1	1	2	-0.5	-0.8	1	1
y	0.25	0.25	0.25	0.25	0.25	0.25	0.5	0.75

❖ REFERENCES:

1. Atul Kumar Singh, Ajay Kumar Sing and Singh, N. P : Heat and Mass transfer flow of aviscous fluid past a vertical plate under oscillatory suction velocity., Ind. J. Pure and Appl.Math., V.34(3),pp.429-442(2003)
2. Lee. T. S., Parikh. P.G. , Archivos. A and Bershader. D : Int. J. Heat Mass Transfer,v.25, pp.499-522(1982)
3. Miyatake.O and Fujii.T : Heat Transfer Jap.Res.,,V.3,pp.29-33(1974)
4. Nelson .D.J and Wood ,B.J : Combined heat and mass transfer natural convection between vertical plates ., Int. J. Heat Mass Transfer,v.32,pp.1789-1792(1989)
5. Nithurasu.K.N., Seetharamu and Sundara Rajan.T : Natural convection heat transfer in a fluid saturated variable porosity medium, Int. J. Heat Mass Transfer.,V.40,No.16, pp. 3955-3967(1997)
6. Singh. K.D., Verma. G.N: Three-dimensional oscillatory flow through a porous medium with periodic permeability,ZAMM,V.8 ,pp.599-604(1995)
7. Singh.K.D., Khem Chand and Verma ,G.N : Heat transfer in a three-dimensionalflow through a porous medium with periodic permeability , ZAMM , V.12, pp.950-952(1995)
8. Singh,K.D and Rakesh Sharma : Three – dimensional free convective flow and mass transfer through a porous medium with periodic permeability.,Ind.J.Pure and Appl. Math.,V.33(6),pp.941-949(2002)
9. Sparrow,E.M,Chrysler,.M and Azvedo. L.F.A : J. Heat Transfer ,V.106, pp.325-332(1984)
10. Trevison.D.V and Bejan , A : Combined heat and mass transfer by natural convection in vertical enclosure ., Trans.ASME,V.109,pp.104-111(1987)
11. Wei-Mon Yan : Int.J.Heat Mass Transfer,V.13,pp.1857-1866(1994)
12. Wei-Mon Yan : Combined buoyancy effects of thermal and mass diffusion on laminar forced convection in horizontal rectangular ducts ., Int.J.Heat Mass Transfer,V.39,No.1,pp.1479-1488(1996)
13. Yan,W.M and Lin,T.F : Combined heat and mass transfer in laminar forced convection channel flows ., Int.Comm.Heat Mass transfer,V.15,pp.333-343(1988).