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GENERALIZED SEMI PRE CONTINUOUS MAPPING IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract: In this manuscript, we inaugurate Neutrosophic generalized semi pre continuous mapping, Neutrosophic generalized semi pre irresolute mapping and Neutrosophic generalized semi pre compact space. We investigate its properties. Also, we add some improvisation of neutrosophic generalized semi pre continuous mapping.
Keywords: Neutrosophic generalized semi pre closed sets, Neutrosophic generalized semi pre continuous mapping, Neutrosophic generalized semi pre irresolute mapping , Neutrosophic generalized semi pre compact space.

1. Introduction

In 2014, Salama, Smarandache and Valeri [9] initiate idea of Neutrosophic closed sets and Neutrosophic continuous functions. Salama, Alblowi [11] launch the conceptualization of generalized Neutrosophic set and generalized Neutrosophic topological spaces. Wadel and Smarandache [14] popularized NOS. Ishwarya and Bageerathi [8] introduced the view of NSO sets in Neutrosophic topological spaces. Rajeshwaran N and Chandramathi N [25] introduced the Neutrosophic generalized semi pre closed sets in Neutrosophic topological spaces. This manuscript we establish Neutrosophic generalized semi pre continuous mapping.. We study their concepts of Neutrosophic generalized semi pre continuous mapping.

2. Preliminaries

Definition 2.1: [10] A neutrosophic topology (NT for short) a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms

(NT1) $0_N, 1_N \in \tau_N$
(NT2) $G_1 \cap G_2 \in \tau_N$
(NT3) $\cup G_i \in \tau_N$, $\forall\{G_i : i \in J\} \subseteq \tau_N$

Here (X_1, τ_N) is called a neutrosophic topological space (NTS for short).

Definition 2.2: [10] Let A_1 *and* A_2 be two Neutrosophic Sets (NS for Short) of the form

$A_1 = \{\langle X, \mu_{A_1}(X), \sigma_{A_1}(X), \gamma_{A_1}(X)\rangle : x \epsilon X\}$, $A_2 = \{\langle X, \mu_{A_2}(X), \sigma_{A_2}(X), \gamma_{A_2}(X)\rangle : x \epsilon X\}$.

(a) $A_1 \subseteq A_2$ if and only if $\mu_{A_1}(X) \leq \mu_{A_2}(X), \sigma_{A_1}(X) \leq \sigma_{A_2}(X)$ and $\gamma_{A_1}(X) \geq \gamma_{A_2}(X)$ for all $x \in X$

(b) ${A_1}^C = \{\langle X, \gamma_{A_1}(X), 1 - \sigma_{A_1}(X), \mu_{A_1}(X)\rangle : x \epsilon X\}$

(c) $A_1 \cap A_2 = \{\langle X, \mu_{A_1}(X) \wedge \mu_{A_2}(X), \sigma_{A_1}(X) \wedge \sigma_{A_2}(X), \gamma_{A_1}(X) \vee \gamma_{A_2}(X)\rangle : x \epsilon X\}$

(d) $A_1 \cup A_2 = \{\langle X, \mu_{A_1}(X) \vee \mu_{A_2}(X), \sigma_{A_1}(X) \vee \sigma_{A_2}(X), \gamma_{A_1}(X) \wedge \gamma_{A_2}(X)\rangle : x \epsilon X\}$

We can use the symbol $A_1 = \{\langle X, \mu_A(X), \sigma_A(X), \gamma_A(X)\rangle : x \epsilon X\}$

The Neutrosophic Sets define by $0_N = \{\langle X, 0,0,1\rangle : x \in X\}$ and $1_N = \{\langle X, 1,1,0\rangle : x \in X\}$.

Definition 2.3: [10] Let (X, τ) be an NTS and $A = \langle x, \mu_A, \sigma_A, \gamma_A\rangle$ be an NTS in X. Then the neutrosophic interior and an neutrosophic closure are defined by

$int(A) = \cup \{G/G$ is a NOS in X and $G \subseteq A\}$.

$cl(A) = \cap \{K/K$ is a NCS in X and $A \subseteq K\}$.

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Definition 2.4: [25] Let (X, τ_N) be a neutrosophic topological space. A subset Ą of (X, τ_N) is called Neutrosophic generalized semi pre closed [NǴŚP -closed] set if $spcl_N$ (Ą) ⊆ Ư , whenever Ą ⊆ Ư and Ư is Neutrosophic open set.
Complement of Neutrosophic generalized semi pre closed set is called the Neutrosophic generalized semi pre [NǴŚP -open] open set.
Definition 2.5:[26] A Neutrosophic topological space (X, τ_N) is said to be NǴŚP normal if for any pair of disjoint NǴŚP closed sets Ą and Ƀ, there exist disjoint Neutrosophic open sets M and N such that Ą ⊂ Ư,Ƀ ⊂ Ṿ.
Preposition 2.5: [13] Let (X_1, τ_N) , (X_2, σ_N) be two neutrosophic topological spaces, if
$g: (X_1, \tau_N) \to (X_2, \sigma_N)$ neutrosophic continuous then it is N- continuous
Definition 2.6: [13] A neutrosophic topological space (X_1, τ_N) is said to be neutrosophic T1/2 if any Neutrosophic closed set in (X_1, τ_N) is neutrosophic closed in (X_1, τ_N)
Definition 2.7: [22] **A** mapping $g: (X_1, \tau_N) \to (X_2, \sigma_N)$ is defined Neutrosophic semi continuous mapping if g^{-1} (K) is NSOS in (X_1, τ_N) for each NOS K (X_2, σ_N).

3. **Neutrosophic Generalized Semi Pre-Continuous mapping**

In this section, we Introduce Neutrosophic generalized semi pre continuous mapping and investigate some of its properties.
Definition 3.1.1: A mapping $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ is called Neutrosophic generalized semi pre continuous (NǴŚP continuous for short) mapping if $\varphi_N^{-1}(Ɗ)$ is NǴŚPĆŚ in (X, τ_N) for every NĆŚ Ɗ of (Y, σ_N).
Example 3.1.2: Let $X = \{a, b, c\}$, $Y = \{u, v, w\}$ and
$K_1 = \{x, \langle 0.2,0.4,0.5\rangle, \langle 0.4,0.5,0.5\rangle, \langle 0.2,0.5,0.2\rangle\}$, $K_2 = \{y, \langle 0.3,0.2,0.3\rangle, \langle 0.4,0.5,0.5\rangle, \langle 0.4,0.4,0.5\rangle\}$
Then $\tau_N = \{0_N, 1_N, K_1\}$ and $\sigma_N = \{0_N, 1_N, K_2\}$ are NTS on (X, τ_N) and (Y, σ_N) respectively. Define a mapping $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ by $\varphi_N(a) = u, \varphi_N(b) = v, \varphi_N(c) = w$. Here the neutrosophic set $K_2^C = \{y, \langle 0.3,0.8,0.3\rangle, \langle 0.5,0.5,0.4\rangle, \langle 0.5,0.6,0.4\rangle\}$ is a neutrosophic closed set in (Y, σ_N). Then
$\varphi_N^{-1}(K_2^C) = \{x, \langle 0.3,0.8,0.3\rangle, \langle 0.5,0.5,0.4\rangle, \langle 0.5,0.6,0.4\rangle\}$ is NǴŚPĆŚ in (X, τ_N). Hence φ_N is NǴŚP continuous mapping.
Theorem 3.1.3: Every Neutrosophic continuous mapping is a NGSP continuous mapping but not conversely.
Proof: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be Neutrosophic continuous mapping. Let Ɗ be a NĆŚ in (Y, σ_N). Then $\varphi_N^{-1}(Ɗ)$ is an NĆŚ in (X, τ_N). Since every NĆŚ is a NǴŚPĆŚ, $\varphi_N^{-1}(Ɗ)$ is a NǴŚPĆŚ in (X, τ_N). Hence φ_N is a NǴŚP continuous mapping
Example 3.1.4: Let $X = \{a, b\}$, $Y = \{u, v\}$ and and $K_1 = \{x, \langle 0.6,0.3,0.2\rangle, \langle 0.5,0.2,0.3\rangle\}$, $K_2 = \{y, \langle 0.5,0.4,0.3\rangle, \langle 0.4,0.2,0.3\rangle\}$. Then $\tau_N = \{0_N, 1_N, K_1\}$ and $\sigma_N = \{0_N, 1_N, K_2\}$ are NTS on (X, τ_N) and (Y, σ_N) respectively. Define a mapping $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ by $\varphi_N(a) = u, \varphi_N(b) = v$. Here the neutrosophic set $K_2^C = \{y, \langle 0.3,0.6,0.5\rangle, \langle 0.3,0.8,0.4\rangle\}$ is a neutrosophic closed set in (Y, σ_N). Then $\varphi_N^{-1}(K_2^C) = \{x, \langle 0.3,0.6,0.5\rangle, \langle 0.3,0.8,0.4\rangle\}$ is NǴŚPĆŚ in (X, τ_N). Hence φ_N is a NǴŚP continuous mapping. But φ_N is not neutrosophic continuous mapping since K_2^C is neutrosophic closed set in (Y, σ_N) but $\varphi_N^{-1}(K_2^C)$ is not a neutrosophic closed set in (X, τ_N) as $cl_N\left(\varphi_N^{-1}(K_2^C)\right) = 1_N \neq \varphi_N^{-1}(K_2^C)$.
Theorem 3.1.5: Every NǴ continuous mapping is a NǴŚP continuous mapping but not conversely.

Proof: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a NG continuous mapping. Let D be a NCŚ in (Y, σ_N). Then $\varphi_N^{-1}(D)$ is a NGCŚ in (X, τ_N). Since every NGCŚ is a NGŚPCŚ, $\varphi_N^{-1}(D)$ is a NGŚPCŚ in (X, τ_N). Hence φ_N is a NGŚP continuous mapping.

Example 3.1.6: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $K_1 = \{x, \langle 0.4,0.4,0.6\rangle, \langle 0.5,0.4,0.5\rangle\}$, $K_2 = \{y, \langle 0.7,0.6,0.2\rangle \langle 0.5,0.6,0.3\rangle\}$. Then $\tau_N = \{0_N, 1_N, K_1\}$ and $\sigma_N = \{0_N, 1_N, K_2\}$ are NTŚ on (X, τ_N) and (Y, σ_N) respectively. Define a mapping $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ by $\varphi_N(a) = u, \varphi_N(b) = v$. Here the neutrosophic set $K_2^C = \{y, \langle 0.2,0.4,0.7\rangle \langle 0.3,0.4,0.5\rangle\}$ is a neutrosophic closed set in (Y, σ_N) . Then $\varphi_N^{-1}(K_2^C) = \{x, \langle 0.2,0.4,0.7\rangle \langle 0.3,0.4,0.5\rangle\}$ is NGŚPCŚ in (X, τ_N). Hence φ_N is a NGŚP continuous mapping. But φ_N is not neutrosophic generalized continuous mapping since K_2^C is neutrosophic closed set in (Y, σ_N) but $\varphi_N^{-1}(K_2^C)$ is not a NGCŚ in (X, τ_N) as $cl_N\left(\varphi_N^{-1}(K_2^C)\right) = K_1^C$.

Theorem 3.1.7: Every NŚ continuous mapping is a NGŚP continuous mapping but not conversely.

Proof: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a NŚ continuous mapping. Let D be a NCŚ in (Y, σ_N). Then $\varphi_N^{-1}(D)$ is a NŚCŚ in (X, τ_N) . Since every NŚCŚ is a NGŚPCŚ, $\varphi_N^{-1}(D)$ is a NGŚPCŚ in (X, τ_N) . Hence φ_N is a NGŚP continuous mapping.

Example 3.1.8: Let $X = \{a, b, c\}$, $Y = \{u, v, w\}$ and $K_1 = \{x, \langle 0.2,0.4,0.6\rangle, \langle 0.1,0.7,0.9\rangle, \langle 0.3,0.6,0.9\rangle\}$, $K_2 = \{y, \langle 0.1,0.2,0.1\rangle, \langle 0.4,0.7,0.6\rangle, \langle 0.3,0.7,0.9\rangle\}$. Then $\tau_N = \{0_N, 1_N, K_1\}$ and $\sigma_N = \{0_N, 1_N, K_2\}$ are NTŚ on (X, τ_N) and (Y, σ_N) respectively. Define a mapping $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ by $\varphi_N(a) = u, \varphi_N(b) = v, \varphi_N(c) = w$. Here the neutrosophic set $K_2^C = \{y, \langle 0.1,0.8,0.1\rangle \langle 0.6,0.3,0.4\rangle \langle 0.9,0.3,0.3\rangle\}$ is a neutrosophic closed set in (Y, σ_N) . Then $\varphi_N^{-1}(K_2^C) = \{x, \langle 0.1,0.8,0.1\rangle \langle 0.6,0.3,0.4\rangle \langle 0.9,0.3,0.3\rangle\}$ is NGŚPCŚ in (X, τ_N). Hence φ_N is a NGŚP continuous mapping. But φ_N is not neutrosophic generalized continuous mapping Then φ_N is NGŚP continuous mapping but not a NŚ continuous mapping. Since K_2^C is a NCŚ in (Y, σ_N) and $\varphi_N^{-1}(K_2^C)$ is not a NŚCŚ in (X, τ_N) Since $int_N\left(cl_N\left(\varphi_N^{-1}(K_2^C)\right)\right) = 1_N \nsubseteq \varphi_N^{-1}(K_2^C)$.

Theorem 3.1.9: Every NP continuous mapping is a NGŚP continuous mapping but not conversely.

Proof: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a NP continuous mapping. Let D be a NCŚ in (Y, σ_N). Then $\varphi_N^{-1}(D)$ is a NPCŚ in (X, τ_N). Since every NPCŚ is a NGŚPCŚ, $\varphi_N^{-1}(D)$ is a NGŚPCŚ in (X, τ_N). Hence φ_N is a NGŚP continuous mapping.

Example: 3.1.10: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $K_1 = \{x, \langle 0.2,0.3,0.8\rangle, \langle 0.5,0.3,0.5\rangle\}, K_2 = \{y, \langle 0.3,0.5,0.6\rangle, \langle 0.2,0.4,0.7\rangle\}$. Then $\tau_N = \{0_N, 1_N, K_1\}$ and $\sigma_N = \{0_N, 1_N, K_2\}$ are NT on (X, τ_N) and (Y, σ_N) respectively Define a mapping $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ by $\varphi_N(a) = u, \varphi_N(b) = v$. Here the neutrosophic set $K_2^C = \{y, \langle 0.6,0.5,0.3\rangle \langle 0.7,0.6,0.2\rangle\}$ is a neutrosophic closed set in (Y, σ_N) . Then $\varphi_N^{-1}(K_2^C) = \{x, \langle 0.6,0.5,0.3\rangle \langle 0.7,0.6,0.2\rangle\}$ is NGŚPCŚ in (X, τ_N).Then φ_N is NGŚP continuous mapping but not NP continuous mapping since K_2^C is NCŚ in (Y, σ_N) and $\varphi_N^{-1}(K_2^C)$ is not a NPCŚ in (X, τ_N) .

Theorem: 3.1.11: Every NŚP continuous mapping is a NGŚP continuous mapping but not conversely.

Proof: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a NŚP continuous mapping. Let D be a NCŚ in (Y, σ_N). Then $\varphi_N^{-1}(D)$ is a NŚPCŚ in (X, τ_N) . Since every NŚPCŚ is NGŚPCŚ, $\varphi_N^{-1}(D)$ is a NGŚPCŚ in (X, τ_N) . Hence φ_N is a NGŚP continuous mapping.

Example 3.1.12 : Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$K_1 = \{x, \langle 0.4,0.4,0.6\rangle, \langle 0.4,0.4,0.3\rangle\}$, $K_2 = \{y, \langle 0.4,0.5,0.6\rangle, \langle 0.3,0.4,0.4\rangle\}$. Then $\tau_N = \{0_N, 1_N, K_1\}$ and $\sigma_N = \{0_N, 1_N, K_2\}$ are NTŚ on (X, τ_N) and (Y, σ_N) respectively. Define a mapping $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ by $\varphi_N(a) = u, \varphi_N(b) = v$. Here the neutrosophic set $K_2^C = \{y, \langle 0.6,0.5,0.4\rangle\langle 0.4,0.6,0.3\rangle\}$ is a neutrosophic closed set in (Y, σ_N) . Then $\varphi_N^{-1}(K_2^C) = \{x, \langle 0.6,0.5,0.4\rangle\langle 0.4,0.6,0.3\rangle\}$ is NGŚPCŚ in (X, τ_N). Then φ_N is NGŚP continuous mapping, but not NŚP continuous mapping. Since K_2^C is NCŚ in (Y, σ_N) and $\varphi_N^{-1}(K_2^C)$ is not a NŚPCŚ in (X, τ_N).

Theorem: 3.1.13: Every Nα continuous mapping is a NGŚP continuous mapping but not conversely.

Proof: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a Nα continuous mapping. Let D be a NCŚ in (Y, σ_N). Then $\varphi_N^{-1}(D)$ is a NαCŚ in (X, τ_N). Since every NαCŚ is NGŚPCŚ, $\varphi_N^{-1}(D)$ is a NGŚPCŚ in (X, τ_N). Hence φ_N is a NGŚP continuous mapping.

Example 3.1.14: Let $X = \{a, b\}$, $Y = \{u, v\}$ and
$K_1 = \{x, \langle 0.2,0.1,0.7\rangle, \langle 0.4,0.4,0.7\rangle\}, K_2 = \{y, \langle 0.9,0.8,0.7\rangle, \langle 0.2,0.4,0.6\rangle\}$. Then
$\tau_N = \{0_N, 1_N, K_1\}$ and $\sigma_N = \{0_N, 1_N, K_2\}$ are NTŚ on (X, τ_N) and (Y, σ_N) respectively. Define a mapping $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ by $\varphi_N(a) = u, \varphi_N(b) = v$. Here the neutrosophic set $K_2^C = \{y, \langle 0.7,0.2,0.9\rangle\langle 0.6,0.6,0.2\rangle\}$ is a neutrosophic closed set in (Y, σ_N) . Then $\varphi_N^{-1}(K_2^C) = \{x, \langle 0.7,0.2,0.9\rangle\langle 0.6,0.6,0.2\rangle\}$ is NGŚPCŚ in (X, τ_N). Then φ_N is NGŚP continuous mapping, but not Nα continuous mapping. Since K_2^C is NCŚ in (Y, σ_N) but $\varphi_N^{-1}(K_2^C)$ is not a NαCŚ in (X, τ_N).

Theorem: 3.1.15: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a mapping where $\varphi_N^{-1}(D)$ is a NRCŚ in (X, τ_N) for every NCŚ D in (Y, σ_N). Then φ_N is a NGŚP continuous mapping but not conversely.

Proof: Assume that $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ is a mapping. Let A be a NCŚ in (Y, σ_N). Then $\varphi_N^{-1}(D)$ is a NRCŚ in (X, τ_N), by hypothesis. Since every NRCŚ is NGŚPCŚ, $\varphi_N^{-1}(D)$ is a NGŚPCŚ in (X, τ_N). Hence φ_N is a NGŚP continuous mapping.

Example 3.1.16: Let $X = \{a, b\}$, $Y = \{u, v\}$ and
$K_1 = \{x, \langle 0.5,0.6,0.5\rangle, \langle 0.5,0.4,0.5\rangle\}, K_2 = \{y, \langle 0.5,0.3,0.5\rangle, \langle 0.5,0.7,0.5\rangle\}$. Then
$\tau_N = \{0_N, 1_N, K_1\}$ and $\sigma_N = \{0_N, 1_N, K_2\}$ are NTŚ on (X, τ_N) and (Y, σ_N) respectively. Define a mapping $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ by $\varphi_N(a) = u, \varphi_N(b) = v$. Here the neutrosophic set $K_2^C = \{y, \langle 0.5,0.7,0.5\rangle\langle 0.5,0.3,0.5\rangle\}$ is a neutrosophic closed set in (Y, σ_N) . Then $\varphi_N^{-1}(K_2^C) = \{x, \langle 0.5,0.7,0.5\rangle\langle 0.5,0.3,0.5\rangle\}$ is NGŚPCŚ in (X, τ_N). Then φ_N is NGŚP continuous mapping, but not NR continuous mapping. Since K_2^C is NCŚ in (Y, σ_N) but $\varphi_N^{-1}(K_2^C)$ is not a NRCŚ in (X, τ_N).

Theorem: 3.1.17: Let $\varphi_N: (X_1, \tau_N) \to (X_2, \sigma_N)$ be a NGŚP continuous mapping and let $\Psi_N : (X_2, \sigma_N) \to (X_3, \eta_N)$ be a N continuous mapping, then $\varphi_N \circ \Psi_N: (X_1, \tau_N) \to (X_3, \eta_N)$ is a NGŚP continuous mapping.

Proof: Let D be a NCŚ in (X_3, η_N). Then $\Psi_N^{-1}(D)$ is a NCŚ in (X_2, σ_N), by hypothesis. Since φ_N is a NGŚP continuous mapping, $\varphi_N^{-1}(\Psi_N^{-1}(D))$ is a NGŚPCŚ in (X_1, τ_N). Hence $\varphi_N \circ \Psi_N$ is a NGŚP continuous mapping.

Theorem: 3.1.18: If $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ is a NGŚP continuous mapping , then for each $NP_{(\alpha,\beta,\gamma)}$ of (X, τ_N) and each $A \in \sigma_N$ such that $\varphi_N(NP_{(\alpha,\beta,\gamma)}) \in A$, there exists a NGŚPOŚ B of (X, τ_N) such that $NP_{(\alpha,\beta,\gamma)} \in B$ and $\varphi_N(B) \subseteq A$.

Proof: Let $NP_{(\alpha,\beta,\gamma)}$ be a NP of (X, τ_N) and $A \in \sigma_N$ such that $\varphi_N(NP_{(\alpha,\beta,\gamma)}) \in A$. Put $B = \varphi_N^{-1}(A)$. Then by hypothesis, B is a NGŚPOŚ in (X, τ_N) such that $NP_{(\alpha,\beta,\gamma)} \in B$ and $\varphi_N(B) = \varphi_N(\varphi_N^{-1}(A)) \subseteq A$.

Theorem: 3.1.19: If $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ is a NGŚP continuous mapping, then for each $NP_{(\alpha,\beta,\gamma)}$ of (X, τ_N) and each $A \in \sigma_N$ such that $\varphi_N(NP_{(\alpha,\beta,\gamma)})qA$, there exists a NGŚPOŚ B of (X, τ_N) such that $NP_{(\alpha,\beta,\gamma)}qB$ and $\varphi_N(B) \subseteq A$.

Proof: Let $NP_{(\alpha,\beta,\gamma)}$ be a NP of X and $A \in \sigma_N$ such that $\varphi_N(NP_{(\alpha,\beta,\gamma)})qA$. Put $B = \varphi_N^{-1}(A)$. Then by hypothesis, B is a NGŚPOŚ in (X, τ_N) such that $NP_{(\alpha,\beta,\gamma)}qB$ and $\varphi_N(B) = \varphi_N(\varphi_N^{-1}(A)) \subseteq A$.

Theorem: 3.1.20: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ is a NGŚP continuous mapping. Then φ_N is a NŚP continuous mapping if (X, τ_N) is a $NŚPT_{1/2}$ space.

Proof: Let D be a NCŚ in (Y, σ_N). Then $\varphi_N^{-1}(D)$ is a NGŚPCŚ in (X, τ_N), by hypothesis. Since (X, τ_N) is a $NŚPT_{1/2}$ space, $\varphi_N^{-1}(D)$ is a NŚPCŚ in (X, τ_N). Hence φ_N is a NŚP continuous mapping.

Theorem: 3.1.21: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a NGŚP continuous mapping and let $\Psi_N: (Y, \sigma_N) \to (Z, \eta_N)$ be a NG continuous mapping where (Y, σ_N) is a $NT_{1/2}$ space. Then $\varphi_N \circ \Psi_N : (X, \tau_N) \to (Z, \eta_N)$ is a NGŚP continuous mapping.

Proof: Let D be a NCŚ in (Z, η_N). Then $\Psi_N^{-1}(D)$ is a NGCŚ in (Y, σ_N), by hypothesis. Since (Y, σ_N) is a $NT_{1/2}$ space, $\Psi_N^{-1}(D)$ is a NCŚ in (Y, σ_N). Therefore $\varphi_N^{-1}(\Psi_N^{-1}(D))$ is a NGŚPCŚ in (X, τ_N), by hypothesis. Hence $\varphi_N \circ \Psi_N$ is a NGŚP continuous mapping.

Theorem: 3.1.22: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a mapping from NT (X, τ_N) to (Y, σ_N). Then the following conditions are equivalent if (X, τ_N) and (Y, σ_N) are $NŚPT_{1/2}$ spaces:

(i) φ_N is a NGŚP continuous mapping,

(ii) $\varphi_N^{-1}(B)$ is a NGŚPOŚ in (X, τ_N) for each NOŚ B in (Y, σ_N).

(iii) For every NP $p_{(\alpha,\beta,\gamma)}$ in (X, τ_N) and for every NOŚ B in (Y, σ_N) such that $\varphi_N(NP_{p(\alpha,\beta,\gamma)}) \in B$, there exists a NGŚPOŚ A in (X, τ_N) such that $(p_{(\alpha,\beta,\gamma)}) \in A$ and $\varphi_N(A) \subseteq B$.

Proof: (i)⇔(ii) is obvious, since $\varphi_N^{-1}(A^C) = (\varphi_N^{-1}(A))^C$.

(ii)⇒(iii) Let B be a any NOŚ in (Y, σ_N) and let $(Np_{(\alpha,\beta,\gamma)}) \in (X, \tau_N)$. Given $\varphi_N(Np_{(\alpha,\beta,\gamma)}) \in B$. By hypothesis $\varphi_N^{-1}(B)$ is a NGŚPOŚ in (X, τ_N). Take $A = \varphi_N^{-1}(B)$. Now $Np_{(\alpha,\beta,\gamma)} \in \varphi_N^{-1}(\varphi_N(Np_{(\alpha,\beta,\gamma)}))$. Therefore $\varphi_N^{-1}(\varphi_N(Np_{(\alpha,\beta,\gamma)})) \in \varphi_N^{-1}(B) = A$. This implies $(Np_{(\alpha,\beta,\gamma)}) \in A$ and $\varphi_N(A) = \varphi_N(\varphi_N^{-1}(B)) \subseteq B$.

(iii)⇒(i) Let A be a NCŚ in (Y, σ_N). Then its complement, say $B = A^C$ is a NOŚ in (Y, σ_N). Let $Np_{(\alpha,\beta,\gamma)} \in (X, \tau_N)$ and $\varphi_N(Np_{(\alpha,\beta,\gamma)}) \in B$. Then there exists a NGŚPOŚ, say C in (X, τ_N) such that $Np_{(\alpha,\beta,\gamma)} \in C$ and $\varphi_N(C) \subseteq B$. Now $C \subseteq \varphi_N^{-1}(\varphi_N(C)) \subseteq \varphi_N^{-1}(B)$. Thus $Np_{(\alpha,\beta,\gamma)} \in \varphi_N^{-1}(B)$. Therefore $\varphi_N^{-1}(B)$ is a NGŚPOŚ in (X, τ_N). That is $\varphi_N^{-1}(A^C)$ is a NGŚPOŚ in (X, τ_N) and hence $\varphi_N^{-1}(A)$ is a NGŚPCŚ in (X, τ_N). Thus φ_N is a NGŚP continuous mapping.

Theorem: 3.1.23: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a mapping from NT (X, τ_N) to NT (Y, σ_N). Then the following conditions are equivalent if (X, τ_N) and (Y, σ_N) are $NŚPT_{1/2}$ spaces:

(i) φ_N is a NGŚP continuous mapping,

(ii) For each NP $p_{(\alpha,\beta,\gamma)}$ in (X, τ_N) and for every Neutrosophic neighborhood (NN for short) A of $\varphi_N(Np_{(\alpha,\beta,\gamma)})$, there exists a NGŚPOŚ B in (X, τ_N) such that $(Np_{(\alpha,\beta,\gamma)}) \in B \subseteq \varphi_N^{-1}(A)$.

(iii) For each NP $p_{(\alpha,\beta,\gamma)}$ in (X, τ_N) and for every NN A of $\varphi_N(Np_{(\alpha,\beta,\gamma)})$, there exists a NGŚPOŚ B in (X, τ_N) such that $(Np_{(\alpha,\beta,\gamma)}) \in B$ and $\varphi_N(B) \subseteq A$.

Proof: (i)⇒(ii) Let $(Np_{(\alpha,\beta,\gamma)}) \in (X, \tau_N)$ and let A be a NN A of $\varphi_N(Np_{(\alpha,\beta,\gamma)})$. Then there exists a NOŚ C in (Y, σ_N) such that $\varphi_N(Np_{(\alpha,\beta,\gamma)}) \in C \subseteq A$. Since φ_N is a NGŚP continuous mapping,

$\varphi_N^{-1}(\text{Ć}) = \text{Ƀ}$(say), is a ƝǴŚƤOŚ in $(\text{Ӽ}, \tau_N)$ and $(\text{Ɲ}p_{(\alpha,\beta,\gamma)}) \in \text{Ƀ} \subseteq \varphi_N^{-1}(\text{Ą})$.
(ii)⇒(iii) Let $(\text{Ɲ}p_{(\alpha,\beta,\gamma)}) \in (\text{Ӽ}, \tau_N)$ and let Ą be a ƝƝ of $\varphi_N(\text{Ɲ}p_{(\alpha,\beta,\gamma)})$. Then there exists a ƝǴŚƤOŚ Ƀ in $(\text{Ӽ}, \tau_N)$ such that $(\text{Ɲ}p_{(\alpha,\beta,\gamma)}) \in \text{Ƀ} \subseteq \varphi_N^{-1}(\text{Ą})$, by hypothesis. Therefore $(\text{Ɲ}p_{(\alpha,\beta,\gamma)}) \in \text{Ƀ}$ and $\varphi_N(\text{Ƀ}) \subseteq \varphi_N(\varphi_N^{-1}(\text{Ą})) \subseteq \text{Ą}$.
(iii)⇒(i) Let Ƀ be a ƝOŚ in (Y, σ_N) and let $(\text{Ɲ}p_{(\alpha,\beta,\gamma)}) \in \varphi_N^{-1}(\text{Ƀ})$. Then $\varphi_N(\text{Ɲ}p_{(\alpha,\beta,\gamma)}) \in \text{Ƀ}$. Therefore Ƀ is a ƝƝ of $\varphi_N(\text{Ɲ}p_{(\alpha,\beta,\gamma)})$. Since Ƀ is ƝOŚ, by hypothesis there exists a ƝǴŚƤOŚ Ą in $(\text{Ӽ}, \tau_N)$ such that $(\text{Ɲ}p_{(\alpha,\beta,\gamma)}) \in \text{Ą} \subseteq \varphi_N^{-1}(\varphi_N(\text{Ą})) \subseteq \varphi_N^{-1}(\text{Ƀ})$. Therefore $\varphi_N^{-1}(\text{Ƀ})$ is a ƝǴŚƤOŚ in $(\text{Ӽ}, \tau_N)$. Hence φ_N is a ƝǴŚƤ continuous mapping.

Theorem: 3.1.24: Let $\varphi_N : (\text{Ӽ}, \tau_N) \to (Y, \sigma_N)$ be a mapping from ƝT $(\text{Ӽ}, \tau_N)$ to ƝT (Y, σ_N). Then the following conditions are equivalent if $(\text{Ӽ}, \tau_N)$ is a ƝŚƤ$T_{1/2}$ spaces:
(i) φ_N is a ƝǴŚƤ continuous mapping,
(ii) If Ƀ is a ƝOŚ in (Y, σ_N) then $\varphi_N^{-1}(\text{Ƀ})$ is a ƝǴŚƤOŚ in $(\text{Ӽ}, \tau_N)$,
(iii) $\varphi_N^{-1}(int_N(\text{Ƀ})) \subseteq cl_N(int_N\left(cl_N\left(\varphi_N^{-1}(\text{Ƀ})\right)\right))$ for every ƝŚ Ƀ in (Y, σ_N) .

Proof: (i)⇔(ii) is obviously true by Theorem 3.1.22
(ii)⇒(iii) Let Ƀ be any ƝŚ in (Y, σ_N). Then $int_N(\text{Ƀ})$ is a ƝOŚ in (Y, σ_N). Then $\varphi_N^{-1}(int_N(\text{Ƀ}))$ is a ƝǴŚƤOŚ in $(\text{Ӽ}, \tau_N)$. Since $(\text{Ӽ}, \tau_N)$ is a ƝŚƤ$T_{1/2}$ space, $\varphi_N^{-1}(int_N(\text{Ƀ}))$ is a ƝŚƤOŚ in $(\text{Ӽ}, \tau_N)$. Therefore
$\varphi_N^{-1}(int_N(\text{Ƀ})) \subseteq cl_N(int_N\left(cl_N\left(\varphi_N^{-1}(int_N(\text{Ƀ}))\right)\right)) \subseteq cl_N(int_N\left(cl_N\left(\varphi_N^{-1}(\text{Ƀ})\right)\right))$.
(iii)⇒(i) Let Ƀ be a ƝOŚ in (Y, σ_N) . By hypothesis $\varphi_N^{-1}(\text{Ƀ}) = \varphi_N^{-1}(int_N(\text{Ƀ})) \subseteq cl_N(int_N\left(cl_N\left(\varphi_N^{-1}(\text{Ƀ})\right)\right))$. This implies $\varphi_N^{-1}(\text{Ƀ})$ is ƝβOŚ in $(\text{Ӽ}, \tau_N)$. Therefore it is a ƝǴŚƤOŚ in $(\text{Ӽ}, \tau_N)$,. and hence φ_N is a ƝǴŚƤ continuous mapping, by Theorem 3.1.22

Theorem: 3.1.25: Let $\varphi_N : (\text{Ӽ}, \tau_N) \to (Y, \sigma_N)$ be a mapping from ƝT $(\text{Ӽ}, \tau_N)$ to ƝT (Y, σ_N). Then the following conditions are equivalent if $(\text{Ӽ}, \tau_N)$ and $(\text{Ӽ}, \tau_N)$ are ƝŚƤ$T_{1/2}$ spaces:
(i) φ_N is a ƝǴŚƤ continuous mapping,
(ii) $int_N\left(cl_N(int_N\left(\varphi_N^{-1}(\text{Ƀ})\right)\right) \subseteq \varphi_N^{-1}(spcl_N(\text{Ƀ}))$ for each ƝĆŚ Ƀ in (Y, σ_N),
(iii) $\varphi_N^{-1}(spint_N(\text{Ƀ})) \subseteq cl_N(int_N(cl_N(\varphi_N^{-1}(\text{Ƀ}))))$ for each ƝOŚ Ƀ of (Y, σ_N),
(iv) $\varphi_N(int_N\left(cl_N(int_N((\text{Ą})))\right)) \subseteq cl_N(\varphi_N(\text{Ą}))$ for each ƝŚ Ą of $(\text{Ӽ}, \tau_N)$.

Proof: (i)⇒(ii) Let Ƀ be a ƝĆŚ in (Y, σ_N). Then $\varphi_N^{-1}(\text{Ƀ})$ is a ƝǴŚƤĆŚ in $(\text{Ӽ}, \tau_N)$. Since $(\text{Ӽ}, \tau_N)$ is a ƝŚƤ$T_{1/2}$ space, $\varphi_N^{-1}(\text{Ƀ})$ is a ƝŚƤĆŚ. Therefore $int_N\left(cl_N(int_N\left(\varphi_N^{-1}(\text{Ƀ})\right)\right) \subseteq \varphi_N^{-1}(\text{Ƀ}) = \varphi_N^{-1}(spcl_N(\text{Ƀ}))$.
(ii)⇒(iii) can be easily proved by taking complement in (ii).
(iii)⇒(iv) Let $\text{Ą} \in (\text{Ӽ}, \tau_N)$. Taking $\text{Ƀ} = \varphi_N(\text{Ą})$ we have $\text{Ą} \subseteq \varphi_N^{-1}(\text{Ƀ})$. Here $int_N\left(\varphi_N(\text{Ą})\right) = int_N(\text{Ƀ})$ is a ƝOŚ in (Y, σ_N) . Then (iii) implies that $\varphi_N^{-1}(spint_N(int_N(\text{Ƀ})) \subseteq cl_N(int_N(cl_N(\varphi_N^{-1}(int_N\text{Ƀ})))) \subseteq cl_N(int_N(cl_N(\varphi_N^{-1}(\text{Ƀ}))))$. Now we have $cl_N(int_N(cl_N(\text{Ą}^C)))^C \subseteq cl_N(int_N(cl_N(\varphi_N^{-1}(\text{Ƀ}^C)))^C \subseteq \varphi_N^{-1}(spint_N(int_N(\text{Ƀ}^C)))^C$. This implies $int_N\left(cl_N(int_N((\text{Ą}))\right) \subseteq \varphi_N^{-1}(spcl_N\left(cl_N(\text{Ƀ})\right))$. Therefore $\varphi_N\left(int_N\left(cl_N(int_N((\text{Ą}))\right)\right) \subseteq \varphi_N\left(\varphi_N^{-1}\left(spcl_N\left(cl_N(\text{Ƀ})\right)\right)\right) \subseteq cl_N(\text{Ƀ}) = cl_N(\varphi_N(\text{Ą}))$.

(iv)⇒(i) Let Ƀ be a any ƝĆŚ in (Y,σ_N). Then $\varphi_N{}^{-1}(Ƀ)$ is a ƝŚ in (X,τ_N) . By hypothesis $\varphi_N\left(\mathrm{int}_N\left(\mathrm{cl}_N(\mathrm{int}_N\left((\varphi_N{}^{-1}(Ƀ))\right))\right)\right) \subseteq \mathrm{cl}_N(\varphi_N(\varphi_N{}^{-1}(Ƀ))) \subseteq \mathrm{cl}_N(Ƀ) = Ƀ$. Now $\mathrm{int}_N(\mathrm{cl}_N(\mathrm{int}_N(\varphi_N{}^{-1}(Ƀ)))) \subseteq \varphi_N{}^{-1}(\varphi_N(\mathrm{int}_N(\mathrm{cl}_N(\mathrm{int}_N(\varphi_N{}^{-1}(Ƀ)))) \subseteq \varphi_N{}^{-1}(Ƀ)$. This implies $\varphi_N{}^{-1}(Ƀ)$ is a ƝβĆŚ and hence it is a ƝǴŚƤĆŚ in (X,τ_N). Thus φ_N is a ƝǴŚƤ continuous mapping.

Theorem: 3.1.26: A mapping $\varphi_N : (X,\tau_N) \to (Y,\sigma_N)$ is a ƝǴŚƤ continuous mapping if $\mathrm{cl}_N(\mathrm{int}_N(\mathrm{cl}_N(\varphi_N{}^{-1}(Ą)))) \subseteq \varphi_N{}^{-1}(\mathrm{cl}_N(Ą))$ for every ƝŚ Ą in (Y,σ_N).
Proof: Let Ą be a ƝOŚ in (Y,σ_N). Then $Ą^C$ is a ƝĆŚ in (Y,σ_N).Therefore $\mathrm{cl}_N(Ą^C) = Ą^C$. By hypothesis,
$\mathrm{cl}_N(\mathrm{int}_N(\mathrm{cl}_N(\varphi_N{}^{-1}(Ą^C)))) \subseteq \varphi_N{}^{-1}\left(\mathrm{cl}_N(Ą^C)\right) = \varphi_N{}^{-1}(Ą^C)$. Now $(\mathrm{int}_N(\mathrm{cl}_N(\mathrm{int}_N(\varphi_N{}^{-1}(Ą)))))^C = \mathrm{cl}_N(\mathrm{int}_N(\mathrm{cl}_N(\varphi_N{}^{-1}(Ą^C)))) \subseteq \varphi_N{}^{-1}(Ą^C) = (\varphi_N{}^{-1}(Ą))^C$. This implies $\varphi_N{}^{-1}(Ą) \subseteq \mathrm{int}_N(\mathrm{cl}_N(\mathrm{int}_N(\varphi_N{}^{-1}(Ą))))$. Hence $\varphi_N{}^{-1}(Ą)$ is a ƝαÓŚ in (X,τ_N) and hence it is a ƝǴŚƤOŚ in (X,τ_N). Therefore φ_N is a ƝǴŚƤ continuous mapping, by Theorem 3.1.22.
Theorem: 3.1.27: Let $\varphi_N : (X,\tau_N) \to (Y,\sigma_N)$ be a mapping from ƝT (X,τ_N) to ƝT (Y,σ_N). Then the following conditions are equivalent if (X,τ_N) is a ƝŚƤ$T_{1/2}$ spaces:

(i) φ_N is a ƝǴŚƤ continuous mapping,

(ii) $\varphi_N{}^{-1}(Ƀ)$ is a ƝǴŚƤĆŚ in (X,τ_N) for every ƝĆŚ Ƀ in (Y,σ_N),

(iii) $\mathrm{int}_N(\mathrm{cl}_N(\mathrm{int}_N(\varphi_N{}^{-1}(Ą)))) \subseteq \varphi_N{}^{-1}(\mathrm{cl}_N(Ą))$ for every ƝŚ Ą in (Y,σ_N).

Proof: (i)⇔(ii) is obviously true by Definition 3.1.1
(ii)⇒(iii) Let Ą be a ƝŚ in (Y,σ_N). Then $\mathrm{cl}_N(Ą)$ is a ƝĆŚ in (Y,σ_N).. By hypothesis, $\varphi_N{}^{-1}(\mathrm{cl}_N(Ą))$ is a ƝǴŚƤĆŚ in (X,τ_N). Since (X,τ_N) is a ƝŚƤ$T_{1/2}$ space, $\varphi_N{}^{-1}(\mathrm{cl}_N(Ą))$ is a ƝŚƤĆŚ in (X,τ_N). Therefore we have $\mathrm{int}_N(\mathrm{cl}_N(\mathrm{int}_N(\varphi_N{}^{-1}(\mathrm{cl}_N(Ą))))) \subseteq \varphi_N{}^{-1}(\mathrm{cl}_N(Ą))$. Now $\mathrm{int}_N(\mathrm{cl}_N(\mathrm{int}_N(\varphi_N{}^{-1}(Ą)))) \subseteq \mathrm{int}_N(\mathrm{cl}_N(\mathrm{int}_N(\varphi_N{}^{-1}(\mathrm{cl}_N(Ą))))) \subseteq \varphi_N{}^{-1}(\mathrm{cl}_N(Ą))$.
(iii)⇒(i) Let Ą be a ƝĆŚ in (Y,σ_N). By hypothesis $\mathrm{int}_N\left(\mathrm{cl}_N(\mathrm{int}_N(\varphi_N{}^{-1}(Ą)))\right) \subseteq \varphi_N{}^{-1}\left(\mathrm{cl}_N(Ą)\right) = \varphi_N{}^{-1}(Ą)$. This implies $\varphi_N{}^{-1}(Ą)$ is a ƝβĆŚ in (X,τ_N) and hence it is a ƝǴŚƤĆŚ. Thus φ_N is a ƝǴŚƤ continuous mapping.

3.2 NEUTROSOPHIC GENERALIZED SEMI PRE IRRESOLUTE MAPPING

In this part, we endeavor to offer an extensive view of ƝǴŚƤ irresolute mappings in Neutrosophic topological spaces, outlining their theoretical bases, fundamental attributes.
Definition 3.2.1: A map $\varphi_N : (X,\tau_N) \to (Y,\sigma_N)$ is called a Neutrosophic generalized semi-pre irresolute (ƝǴŚƤ irresolute for short) mapping if $\varphi_N{}^{-1}(Ɗ)$ is a ƝǴŚƤ closed sets in (X,τ_N) for every ƝǴŚƤ closed set Ɗ of (Y,σ_N).
Example 3.2.2: Let $X = \{a,b\}$, $Y = \{u,v\}$ and
$K_1 = \{x, \langle 0.5,0.6,0.1\rangle, \langle 0.5,0.4,0.1\rangle\}$, $K_2 = \{y, \langle 0.5,0.3,0.2\rangle, \langle 0.5,0.7,0.2\rangle\}$. Then $\tau_N = \{0_N, 1_N, K_1\}$ and $\sigma_N = \{0_N, 1_N, K_2\}$ are ƝTŚ on (X,τ_N) and (Y,σ_N) respectively. Define a mapping $\varphi_N : (X,\tau_N) \to (Y,\sigma_N)$ by $\varphi_N(a) = u$, $\varphi_N(b) = v$. Here the neutrosophic set $K_3 = \{y, \langle 0.3,0.3,0.7\rangle, \langle 0.2,0.3,0.7\rangle\}$ is a ƝǴŚƤ closed set in (Y,σ_N). Then φ_N is a ƝǴŚƤ irresolute mapping since $\varphi_N{}^{-1}(K_3)$ is ƝǴŚƤ closed set in (X,τ_N).
Theorem: 3.2.3: If $\varphi_N : (X,\tau_N) \to (Y,\sigma_N)$ is a ƝǴŚƤ irresolute mapping then φ_N is a ƝǴŚƤ continuous mapping but not conversely.

Proof: Let φ_N be a NGŚP irresolute mapping. Let Ɗ be any Neutrosophic closed sets in (Y, σ_N). Then Ɗ is a NGŚP closed sets and by hypothesis ${\varphi_N}^{-1}(Ɗ)$ is a NGŚP closed sets in (X, τ_N). Hence φ_N is a NGŚP continuous mapping.

Example 3.2.4: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $K_1 = \{x, \langle 0.4,0.3,0.6\rangle, \langle 0.5,0.3,0.5\rangle\}$, $K_2 = \{y, \langle 0.6,0.7,0.3\rangle, \langle 0.7,0.8,0.3\rangle\}$. Then $\tau_N = \{0_N, 1_N, K_1\}$ and $\sigma_N = \{0_N, 1_N, K_2\}$ are NTŚ on (X, τ_N) and (Y, σ_N) respectively. Define a mapping $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ by $\varphi_N(a) = u$, $\varphi_N(b) = v$. Then φ_N is NGŚP continuous mapping since for a neutrosophic closed set $K_2^C = \{y, \langle 0.3,0.3,0.6\rangle, \langle 0.3,0.2,0.7\rangle\}$ in (Y, σ_N), its inverse image ${\varphi_N}^{-1}(K_2^C)$ is NGŚP closed set in (X, τ_N) . But φ_N is not a NGŚP irresolute mapping. For the neutrosophic set $K_3 = \{y, \langle 0.4,0.3,0.6\rangle, \langle 0.5,0.3,0.5\rangle\}$ is NGŚP closed set in (Y, σ_N) but ${\varphi_N}^{-1}(K_3)$ is not a NGŚP closed set in (X, τ_N).

Theorem: 3.2.5: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ and $\Psi_N : (Y, \sigma_N) \to (Z, \eta_N)$ be a NGŚP irresolute mapping. Then $\varphi_N \circ \Psi_N : (X, \tau_N) \to (Z, \eta_N)$ is a NGŚP irresolute mapping.

Proof: Let Ɗ be a NGŚP closed set in (Z, η_N). Then ${\Psi_N}^{-1}(Ɗ)$ is a NGŚP closed set in (Y, σ_N). Since φ_N is a NGŚP irresolute, ${\varphi_N}^{-1}({\Psi_N}^{-1}(Ɗ))$ is a NGŚP closed set in (X, τ_N), by hypothesis. Hence $\varphi_N \circ \Psi_N$ is a NGŚP irresolute mapping.

Theorem: 3.2.6: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a NGŚP irresolute mapping and $\Psi_N : (Y, \sigma_N) \to (Z, \eta_N)$ be a NGŚP continuous mapping. Then $\Psi_N \circ \varphi_N : (X, \tau_N) \to (Z, \eta_N)$ is a NGŚP continuous mapping.

Proof: Let Ɗ be a Neutrosophic closed set in (Z, η_N). Then ${\Psi_N}^{-1}(Ɗ)$ is a NGŚP closed set in (Y, σ_N). Since φ_N is a NGŚP irresolute mapping, ${\varphi_N}^{-1}({\Psi_N}^{-1}(Ɗ))$ is a NGŚP closed set in (X, τ_N). Hence $\varphi_N \circ \Psi_N$ is a NGŚP continuous mapping.

Theorem: 3.2.7: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a mapping from a NT (X, τ_N) into a NT (Y, σ_N). Then the following conditions are equivalent if (X, τ_N) and (Y, σ_N) are NŚPT$_{1/2}$ spaces:

(i) φ_N is a NGŚP irresolute mapping,

(ii) ${\varphi_N}^{-1}(Ɗ)$ is a NGŚP open set in (X, τ_N) for each NGŚP open set in (Y, σ_N),

(iii) ${\varphi_N}^{-1}(spint_N(Ɗ)) \subseteq spint_N({\varphi_N}^{-1}(Ɗ))$ for each NŚ Ɗ of (Y, σ_N),

(iv) $spcl_N({\varphi_N}^{-1}(Ɗ)) \subseteq {\varphi_N}^{-1}(spcl_N(Ɗ))$ for each NŚ Ɗ of (Y, σ_N).

Proof: (i)⇔(ii) is obvious, since ${\varphi_N}^{-1}(Ƀ^C) = ({\varphi_N}^{-1}(Ƀ))^C$.

(ii)⇒(iii) Let Ɗ be any NŚ in (Y, σ_N) and $spint_N(Ɗ) \subseteq Ɗ$. Also ${\varphi_N}^{-1}(spint_N(Ɗ)) \subseteq {\varphi_N}^{-1}(Ƀ)$. Since $spint_N(Ɗ)$ is a NŚP open set in (Y, σ_N), it is a NGŚP open set in (Y, σ_N). Therefore ${\varphi_N}^{-1}(spint_N(Ɗ))$ is a NGŚP open set in (X, τ_N), by hypothesis. Since (X, τ_N) is a NŚPT$_{1/2}$ space, ${\varphi_N}^{-1}(spint_N(Ɗ))$ is a NŚP open set in (X, τ_N) . Hence ${\varphi_N}^{-1}\left(spint_N(Ɗ)\right)=spint_N({\varphi_N}^{-1}\left(spint_N(Ɗ)\right)) \subseteq spint_N({\varphi_N}^{-1}(Ɗ))$

(iii)⇒(iv) is obvious by taking complement in (iii).

(iv)⇒(i) Let Ɗ be a NGŚP closed set in (Y, σ_N). Since (Y, σ_N) is a NŚPT$_{1/2}$ space, Ɗ is a NŚP closed sets in (Y, σ_N) and $spcl_N(Ɗ) = Ɗ$. Hence ${\varphi_N}^{-1}(Ɗ) = {\varphi_N}^{-1}(spcl_N(Ɗ)) \supseteq spcl_N({\varphi_N}^{-1}(Ɗ))$, by hypothesis. But ${\varphi_N}^{-1}(Ɗ) \subseteq spcl_N({\varphi_N}^{-1}(Ɗ))$. Therefore $spcl_N({\varphi_N}^{-1}(Ɗ)) = {\varphi_N}^{-1}(Ɗ)$. This implies ${\varphi_N}^{-1}(Ɗ)$ is a NŚP closed sets and hence it is a NGŚP closed set in (X, τ_N). Thus φ_N is a NGŚP irresolute mapping.

Theorem: 3.2.8: Let $\varphi_N : (X, \tau_N) \to (Y, \sigma_N)$ be a NGŚP irresolute mapping from a NT (X, τ_N) into (Y, σ_N). Then $\varphi_N^{-1}(D) \subseteq \mathrm{spint}_N(\varphi_N^{-1}(\mathrm{cl}_N(\mathrm{int}_N(\mathrm{cl}_N(D)))))$ for every NGŚP open set D in (Y, σ_N), if (X, τ_N) and (Y, σ_N) are $NŚPT_{1/2}$ spaces.

Proof: Let D be a NGŚP open set in (Y, σ_N). Then by hypothesis $\varphi_N^{-1}(D)$ is a NGŚP open set in (X, τ_N). Since (X, τ_N) is $NŚPT_{1/2}$ space, $\varphi_N^{-1}(D)$ is a NŚP open set in (X, τ_N). Therefore $\mathrm{spint}_N(\varphi_N^{-1}(D)) = \varphi_N^{-1}(D)$. Since (Y, σ_N) is $NŚPT_{1/2}$ space, D is a NŚP open set (Y, σ_N) and $D \subseteq \mathrm{cl}_N(\mathrm{int}_N(\mathrm{cl}_N(D)))$. Now, $\varphi_N^{-1}(D) = \mathrm{spint}_N(\varphi_N^{-1}(D))$, implies
$\varphi_N^{-1}(D) \subseteq \mathrm{spint}_N(\varphi_N^{-1}(\mathrm{cl}_N(\mathrm{int}_N(\mathrm{cl}_N(D)))))$.

3.3 NEUTROSOPHIC GENERALIZED SEMI PRE COMPACT SPACE.

In this segment, our goal is to furnish a thorough exploration of NGŚP compact space within Neutrosophic topological spaces, clarifying their theoretical underpinnings and fundamental characteristics.

Definition: 3.3.1: Let (X, τ_N) be a NTŚ. If a family $\{\langle x, \mu_{G_i}(x), \sigma_{G_i}(x), \gamma_{G_i}(x)\rangle : i \in J\}$ of NGŚP open sets in (X, τ_N) satisfies the condition $\cup \{\langle x, \mu_{G_i}(x), \sigma_{G_i}(x), \gamma_{G_i}(x)\rangle : i \in J\} = 1_N$, then is called a NGŚP open cover of (X, τ_N).

Definition: 3.3.2: Let (X, τ_N) be a NTŚ. A finite subfamily of a NGŚP open cover $\{\langle x, \mu_{G_i}(x), \sigma_{G_i}(x), \gamma_{G_i}(x)\rangle : i \in J\}$ of (X, τ_N), which is also a NGŚP open cover of (X, τ_N) is called a finite subcover of $\{\langle x, \mu_{G_i}(x), \sigma_{G_i}(x), \gamma_{G_i}(x)\rangle : i \in J\}$.

Definition: 3.3.3: A NTŚ (X, τ_N) is called NGŚP compact iff every NGŚP open cover of (X, τ_N) has a finite subcover.

Definition: 3.3.4: Let (X, τ_N) be a NTŚ. A family $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ of NGŚP closed sets in (X, τ_N) satisfies the finite intersection property (in short NIP) iff every finite subfamily $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i = 1,2,\dots,n\}$, of the family satisfies the condition $\cap_{i=1}^{n} \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle\} \neq 0_N$.

Theorem 3.3.5: A NTŚ (X, τ_N) is NGŚP compact iff every family $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ of NGŚP closed sets with finite intersection property has a non empty intersection.

Proof: Let NTŚ (X, τ_N) is NGŚP-compact. Suppose, $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ be any family of NGŚP-closed sets in (X, τ_N) such that $\cap \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\} = 0_N$. Thus, this implies that, $\{\langle x, \wedge\mu_{K_i}(x), \wedge\sigma_{K_i}(x), \vee\gamma_{K_i}(x)\rangle\} = 0_N$, $\{\wedge\mu_{K_i}(x) : i \in J\} = 0$, $\{\wedge\sigma_{K_i}(x) : i \in J\} = 0$ and $\{\vee\gamma_{K_i}(x) : i \in J\} = 1$, $\Rightarrow$ $\cup \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\} = 1_N$. Thus, $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ is a NGŚP open cover of (X, τ_N). Since (X, τ_N) is NGŚP-compact, so every NGŚP-open cover of (X, τ_N) has finite subcover. Therefore, (X, τ_N) has finite subcover $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i = 1,2,\dots,n\}$. So $\cup_{i=1}^{n} \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle\} = 1_N$, this implies that $\{\langle x, \vee_{i=1}^{n}\mu_{K_i}(x), \vee_{i=1}^{n}\sigma_{K_i}(x), \wedge_{i=1}^{n}\gamma_{K_i}(x)\rangle : i \in J\} = 1_N$, $\vee_{i=1}^{n}\{\mu_{K_i}(x)\} = 0$, $\vee_{i=1}^{n}\{\sigma_{K_i}(x)\} = 0$ and $\wedge_{i=1}^{n}\{\gamma_{K_i}(x)\} = 1$ implies that $\cap_{i=1}^{n} \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle\} = 0_N$, which contradicts to our hypothesis. Hence, every family of NGŚP closed set with finite intersection property has a non empty intersection.

Conversely, suppose every family of NGŚP closed set with finite inersecton property has a non empty intersection. Assume that , $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ is any NGŚP open cover of (X, τ_N), then $\cup \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\} = 1_N$. Therefore $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ is a family of NGŚP closed sets in (X, τ_N) such that $\cap \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\} = 0_N$. By

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assumption, we can find a finite subfamily, $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i = 1,2, \dots, n\}$ such that $\cap_{i=1}^{n} \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle\} = 0_N$, which implies, $\cup_{i=1}^{n} \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle\} = 1_N$. Thus $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i = 1,2, \dots, n\} = 1_N$ is a finite subcover of (X, τ_N). Hence (X, τ_N) is ƝĜŚƤ compact.

Definition: 3.3.6: Let (X, τ_N) be a ƝTŚ and Ą be a Neutrosophic set in (X, τ_N). If a family $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ of ƝĜŚƤ open sets in (X, τ_N) satisfies the condition Ą $\subseteq \cup$ $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$, then it is called a ƝĜŚƤ open cover of Ą.

Definition: 3.3.7: Let (X, τ_N) be a ƝTŚ. A finite subfamily of a ƝĜŚƤ open cover $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ of Ą, which is also a ƝĜŚƤ open cover of Ą is called a finite subcover of $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$.

Definition: 3.3.8: The Neutrosophic set Ą $= \langle x, \mu_{Ą}, \sigma_{Ą}, \gamma_{Ą}\rangle$ in a ƝTŚ (X, τ_N) is called ƝĜŚƤ compact iff every ƝĜŚƤ open cover of Ą has a finite subcover.

Theorem 3.3.9: Let (X, τ_N) be a ƝTŚ. A ƝĜŚƤ closed subset of a ƝĜŚƤ compact space is Neutrosophic compact relative to (X, τ_N).

Proof: Let Ą be a ƝĜŚƤ closed subset of (X, τ_N). Let $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ be ƝĜŚƤ open cover of Ą. Then the family $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\} \cup Ą^C$ is ƝĜŚƤ open cover of (X, τ_N) . Since (X, τ_N) is a ƝĜŚƤ compact, there is a finite subfamily $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i = 1,2, \dots, n\}$ of ƝĜŚƤ open cover, which also covers (X, τ_N). If this cover contains $Ą^C$ we discard it. Otherwise leave the subcover as it is . Thus, we obtained a finite ƝĜŚƤ open subcover of Ą. So Ą is ƝĜŚƤ compact relative to (X, τ_N).

Theorem 3.3.10: Let (X, τ_N) be a ƝTŚ. If (X, τ_N) is ƝĜŚƤ compact space, then it is compact.

Proof: Suppose, (X, τ_N) be a ƝĜŚƤ compact. Assume contrary that (X, τ_N) is not fuzzy compact, then there is atleast one fuzzy open cover $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ of (X, τ_N) not has a finite subcover, implies that $\cup \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle\} = 1_N$ a open cover of (X, τ_N) such that $\cup_{i=1}^{n} \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle\} = 1_N$. Since, every Neutrosophic open set is ƝĜŚƤ open set. Therefore a open cover $\{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle : i \in J\}$ of (X, τ_N) becomes ƝĜŚƤ open cover of (X, τ_N) such that $\cup_{i=1}^{n} \{\langle x, \mu_{K_i}(x), \sigma_{K_i}(x), \gamma_{K_i}(x)\rangle\} = 1_N$, which is a contradiction. Hence, if (X, τ_N) is ƝĜŚƤ compact space, then it is compact.

REFERENCE

[1] Atanassov.K. T, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87–96.

[2] Chang.C. L, Fuzzy topological spaces, Journal of Mathematical Analysis and Application, 24(1968), 183–190.

[3] Dhavaseelan.R and Jafari, Generalized Neutrosophic closed sets, new trends in Neutrosophic theory and applications, 2(2018), 261–273.

[4] Dogan Coker, An introduction to intuitionstic fuzzy topological spaces, Fuzzy Sets and Systems, 88(1997), 81–89.

[5] Floretin Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301,USA, 2002.

[6] Floretin Smarandache, Neutrosophic Set:- A Generalization of Intuitionistiic Fuzzy set, Journal of Defense Resourses Management, 1(2010),107–116.

[7] Floretin Smarandache, A Unifying Field in Logic: Neutrosophic Logic. Neutrosophy, Neutrosophic set, Neutrosophic Probability. Ameican Research Press, Rehoboth, NM,1999.

[8] Ishwarya.P and K. Bageerathi, On Neutrosophic semi-open sets in Neutrosophic topological spaces,International Jour. of Math. Trends and Tech. 2016, 214-223.
[9] Salama A.A and, Florentin Smarandache and Valeri Kroumov, Neutrosophic closed sets abs Neutrosophic continuous Function, Neutrosophic Sets and System, 2014,4, 4-8.
[10] Salama A.A and Alblowi S.A, Neutrosophic set and Neutrosophic topological space,ISOR J. Mathematics, 3(4)(2012), 31–35.
[11] Salama.A.A and Alblowi.S.A, Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, Journal computer Sci. Engineering, 2(7)(2012), 12–23.
[12] Shanthi.V.K, Chandrasekar.S and Safina Begam.K, Neutrosophic Generalized Semi Closed sets in Neutrosophic Topological spaces, International Journal of Research in Advent Technology, 6(7)(2018), 2321–9637.
[13] SalamaA.A, Florentin Smarandache and Valeri Kroumov, Neutrosophic Closed set and Neutrosophic Continuous Function, Neutrosophic Sets and Systems,4(2014), 4–8.
[14] Wadel Faris Al-omeri and Florentin Smarandache, New Neutrosophic Sets via Neutrosophic Topological Spaces, New Trends in Neutrosophic Theory and Applications, Vol(2) June 2016.
[15] Zadeh.L.A, Fuzzy set, Inform and Control, 8(1965), 338–353.
[16] Al-Omeri, W.F., and Jafari, S., Neutrosophic pre-continuous multifunctions and almost pre-continuous multifunctions, Neutrosophic Sets and Systems, Vol 27, pp 53-69, 2019.
[17] Vadivel, M. Seenivasan and C. John Sundar, An introduction to δ-open sets in a neutrosophic topological spaces, Journal of Physics: Conference Series, 1724 (2021), 012011.
[18] A. Vadivel and C. John Sundar, Neutrosophic δ-Open Maps and Neutrosophic δ-Closed Maps, International Journal of Neutrosophic Science (IJNS), 13 (2) (2021), 66-74.
[19] A. Vadivel, P. Thangaraja and C. John Sundar, Neutrosophic e-continuous maps and neutrosophic e-irresolute maps, Turkish Journal of Computer and Mathematics Education,12 (1S) (2021), 369-375.
[20] A. Vadivel, P. Thangaraja and C. John Sundar, Neutrosophic e-Open Maps, Neutrosophic e-Closed Maps and Neutrosophic e-Homeomorphisms in Neutrosophic Topological Spaces, AIP Conference Proceedings, 2364 (2021), 020016.
[21] N. Moogambigai, A. Vadivel, and S. Tamilselvan, Neutrosophic Z-continuous maps a and Z-irresolute maps, AIP Conference Proceedings, 2364 (2021), 020020.
[22] Bhimraj Basumatary, Nijwm Wary,Jeevan Krishna Khaklary and Usha Rani Basumatary, On Some Properties of Neutrosophic Semi Continuous and Almost Continuous Mapping,Computer Modeling in Engineering & Sciences, cmes.2022.018066
[23] Gautam Chandra Ray and Sudeep Dey, Relation of Quasi-coincidence for Neutrosophic Sets, Neutrosophic Sets and Systems, Vol. 46, 2022
[24]Charanya,Dr.K.Ramasamy, Pre semi Homeomorphisms and Generalized semi pre Homeomorphisms in Topological spaces, International Journal of Mathematics Trends and Technology (IJMTT) – Volume 42 Number 1- February 2017.
[25] Rajeshwaran N and Chandramathi N , , Government Arts College, Udumalpet, India GENERALIZED SEMI PRE CLOSED SETS VIA NEUTROSOPHIC TOPOLOGICAL SPACE, pages 174-180, **https://doi.org/10.37896/sr8.12/016**
[26] Rajeshwaran N and Chandramathi N, Government Arts College, Udumalpet, India, Neutosophic generalized semi pre regular and normal space, International Journal of Scientific Research in Engineering and Management (IJSREM) on Volume 08, Issue 07 July 2024.