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Cracked functionally graded beam's effects on vibration characteristics

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Abstract: Existing cracks or fractures can occur in technical beam constructions as a natural phenomenon. Because FGM changes its physical and mechanical properties, it is considered a state-of-the-art material class and is used in many structural applications. These structures are exposed to dynamic conditions like other metals and compounds. As a result, the FGM structure with early cracks performs significantly worse when exposed to a dynamic environment. As a result, there is much interest in studying how structures with cracks perform. This study investigated the relationship between the dynamic property as a function of the natural frequency and the initial fracture at different locations of varying severity. For 20mm stainless steel×20mm×this, a 500mm FGM cantilever beam (SUS30: contains 18% Cr and 8% Ni) is simulated in free vibration and the dynamic parameters of the beam are analyzed with a finite element tool. : ANSYS.

Keywords: FGM; Cantilever; crack; finite element; free vibration

1. Introduction

The functionally graded material is originally designed as a high-quality thermally bonded material that meets the requirements of researchers for temperature differences of up to 1000° within a few millimeters of thickness. Currently, there is a continuous material development process, FG materials are designed according to the need. Since the use of aerospace parts in electronic equipment, FG materials have gained widespread popularity and acceptance. However, the initiation and initiation of cracks is a common dynamic process that leads to a reduction of stiffness and thus changes the dynamic characteristics of the structure. The broken issues of FGM are very important and must evaluate the functionality and integrity of the FGM structure.

Rizos et al. [1] proposed a system and investigated the detection of crack location and size based on the vibration modes of a cantilever beam. Shen and Pierre [2] used Bernoulli-Euler beam theory to find natural modes of vibration with symmetrical cracks. Liang et al. [3] proposed a method to detect cracks in beam structures by calculating natural frequencies. Narkis [ ] developed a system to locate cracks in a simply supported beam using an algebraic model validated with FEA results. Krawczuk and Ostachowicz [5] analyzed a cantilever beam made of graphite fiber-reinforced polyimide with a transverse non propagating open crack at its edge using the clamp spring method and finite element methods.

Nandwana and Maiti [6] used a torsional spring model to analyze the internal normal and observed location of a beveled edge or crack in a thin beam based on natural frequency measurements.

Erdogan and Wu [7] studied the surface cracking problem of a plate with functionally graded properties. Hsu [8] used the differential quadrature method to determine natural frequencies and mode shapes of beams. Lin and Chang [9] developed an analytical method to study the dynamic response of a cracked cantilever beam subjected to a concentrated moving load, modeled as a two-span beam, using Euler-Bernoulli beam theory. Loya et al. [10] studied the bending characteristics of a cracked beam.

Kisa and Gurel [11] use this technique to find the natural frequencies of stepped circularly diffracted beams. Aydin [12-13] investigated the vibration characteristics of Timoshenko beams and Euler-Bernoulli beams with any number of cracks. ang et al. [1] used an analytical method to calculate natural frequencies of broken FGM beams based on Euler-Bernoulli theories and a crack spring model. The authors also study the free and forced vibration of homogeneous Euler-Bernoulli beams under axial force and transverse moving load. That et al. [15] investigated the effect of open edge cracks on the vibration of an FGM Timoshenko beam with different boundary conditions. Matbuly et al. [16] investigated the free vibration of elastically supported cracked FGM beams on a Winkler-Pasternak foundation. Kitipornchai et al [17] investigated the dynamic behavior of shear deformations of edge cracks in functionally graded beams under a moving load on an elastic base.

Wei et al. [18] established the equations of motion for cracked beams with rotational inertia and shear deformation. Sherapatnia et al. [19] studied the modes and natural frequencies of cracked beams using Euler-Bernoulli, Rayleigh, shear deformation and Timoshenko theories. Khiem and Huyen [20] proposed a method to detect a single crack in a functionally graded Timoshenko beam at natural frequencies. Jena et al. [20] carried out a modal study of cracked composite beams numerically and confirmed the result with experimental results. Different types of FG materials with different process techniques and applications are developed in different literatures. The purpose of this work is to provide an overview of the FG material concept and processing techniques and material property formulas.

2.Mathematical Model

For the study, an FGM cantilever beam with dimensions $L \times b \times h$ is considered as shown in Figure 1a. The mechanical properties of the FGM beam are varied along the height of the beam according to the power distribution formula as depicted by equation (1).

$$P(z) = [P_t - P_b]\left(\frac{2z+h}{2h}\right)^n + P_b \qquad (1)$$

Where n = no. of layers, z = thickness of each layer, Pt= top layer properties, P_b = bottom layer property, net property of FG beam.

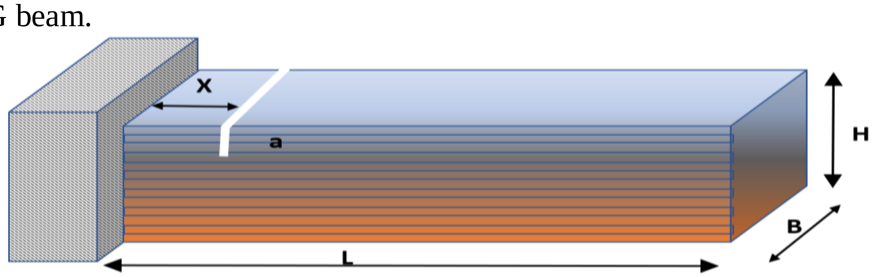

Figure 1:(a) A schematic view of FGM crack beam

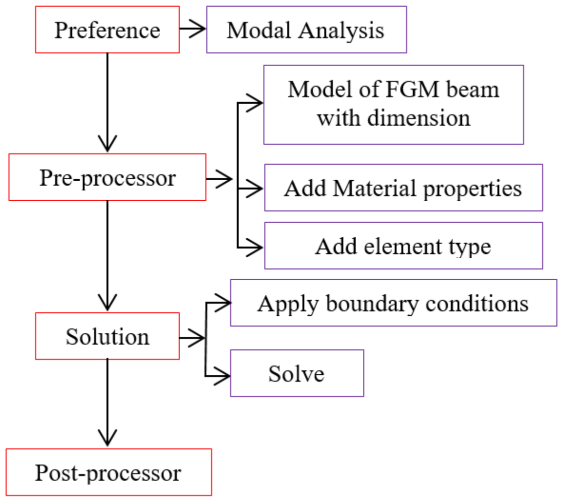

Figure 1: (b) FEA steps in block diagram

The modal characteristics of the composite beam can be incorporated into Castiglione's theorem where the natural frequency and mode shape can be described as a function of the strain energy release rate of a cracked FGM beam given by

$$J = \sum_{i=1}^{n} \frac{1-\nu^2}{E} K_i^2 \qquad (2)$$

where ν, E and K_i are Poisson's ratio, modulus of elasticity, and the stress intensity factors.
The flexibility influence co-efficient of a cracked beam is given by

$$\frac{\partial U_i}{\partial F_j} = \frac{\partial^2}{\partial F_j \partial F_i} \int_0^a (\sum_{i=1}^{n} K)^2 d\xi = C_{ij} \qquad (3)$$

2. Result and discussion

The effect of an open crack on the dynamic behavior of FGM is studied as shown in Figure 2. For this purpose, an FGM beam of size 500mm×20mm×20mm made of SUS304 and Si_3N_4 is taken for the study in the FEA software of ANSYS 14.0. The detailed FEA steps are given in the block diagram (Figure 1b). The result of the cracked beam () with an un-cracked beam is studied and compared. For this, two relative terms, relative cracked position and relative crack severity (RCS) are used.

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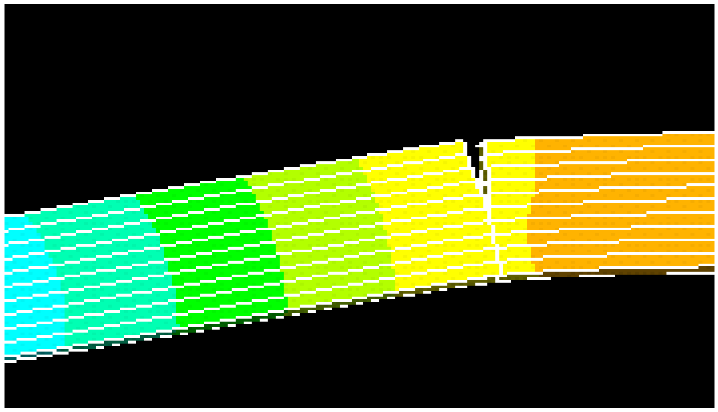

Figure 2:FEA model of a cracked beam

The relative crack position (RCP) and relative crack severity (RCS) are defined as,

$$RCP = \frac{\text{position of the crack}}{\text{total length of the beam}} = \frac{x}{L}$$

$$RCS = \frac{\text{height of the crack}}{\text{total height of the beam}} = \frac{a}{H}$$

In this work, the first four relative natural frequencies (RNF) are the ratio of the natural frequency of a cracked beam to the natural frequency of an un-cracked beam is determined. The variation of RNF with RCP for fixed RCS and the relation of RNF with RCS for fixed RCP is examined and elaborated. For the cantilever beam, the RCPs of 0.05, 0.1, 0.2, 0.4, 0.6, and 0.8 is taken and for the RCSs of 0.1, 0.2, 0.3, 0.4, and 0.5 is considered for the current investigation(see, Figure 3a-3c).

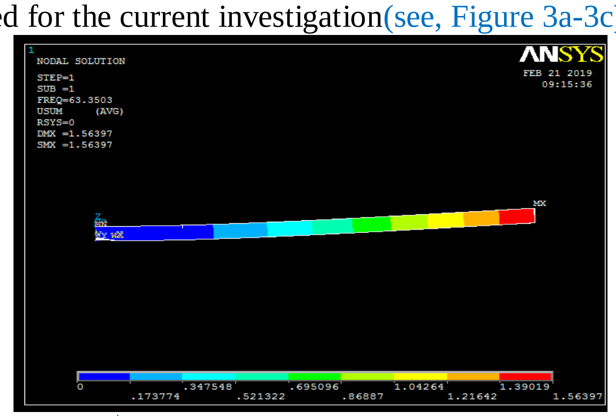

Figure 3:(a)1st mode of vibration with RCP 0.2 & RCS 0.4

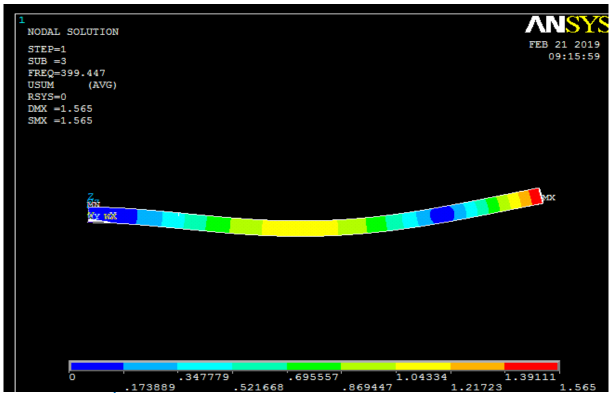

Figure 3:(b)2nd mode of vibration with RCP 0.2 & RCS 0.4

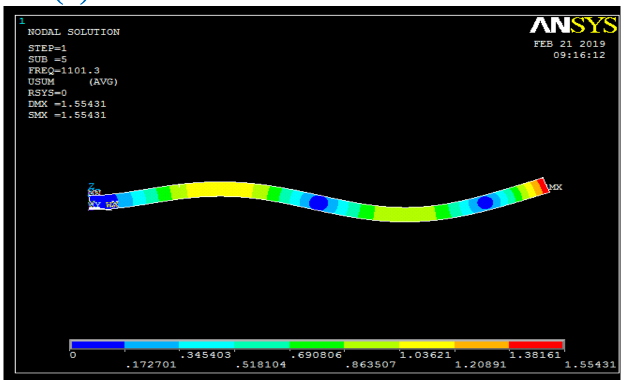

Figure 3:(c)3rd mode of vibration with RCP 0.2 & RCS 0.4

Table 1: RNF of cracked FGM beam

RCP	RNF	RCS				
		0.1	**0.2**	**0.3**	**0.4**	**0.5**
0.05	1	0.990296539	0.97674881	0.948579609	0.900844413	0.831435502
	2	0.992775552	0.984123263	0.967415395	0.941820817	0.909790299
	3	0.994686995	0.98952799	0.97990298	0.965811966	0.949179949
	4	0.996085011	0.993048897	0.987256312	0.977708533	0.963966123
0.1	1	0.986067845	0.974561555	0.950475231	0.908996911	0.847236767
	2	0.991837865	0.987831387	0.979882363	0.967053107	0.94974853
	3	0.995302995	0.994609995	0.993300993	0.991144991	0.988064988
	4	0.996923937	0.996883988	0.996724193	0.996444551	0.995885267
0.2	1	0.97953259	0.971313812	0.954465315	0.92424142	0.877182285
	2	0.991987043	0.99173131	0.991837865	0.991646066	0.9913264
	3	0.994301994	0.991606992	0.986293986	0.976514977	0.961268961
	4	0.993088846	0.986936721	0.974153084	0.949944072	0.910274848
0.4	1	0.977663481	0.978578151	0.967602105	0.954809974	0.927926614
	2	0.996291876	0.997932828	0.978838121	0.957974597	0.918826187
	3	0.991760992	0.992607993	0.982751983	0.972433972	0.94956495
	4	0.998082454	0.998721636	0.99085171	0.981184084	0.95985139
0.6	1	0.956374193	0.956652571	0.955565571	0.951244084	0.945954903
	2	0.958081153	0.960425369	0.951048504	0.915970505	0.877589293
	3	0.963193963	0.964656965	0.959035959	0.939323939	0.91953492
	4	0.964365612	0.964844998	0.962927453	0.955816555	0.947826782
0.8	1	0.970080995	0.970213556	0.970080995	0.969073532	0.969166324
	2	0.970718609	0.969546501	0.966584264	0.960148325	0.950600972
	3	0.972125972	0.966889967	0.955262955	0.932932933	0.897435897
	4	0.974832215	0.966962288	0.950303611	0.921540428	0.882710131

It is quite evident that the increase in relative crack severity at any position decreases the stiffness of the beam and as a result, there is a significant decrease in natural frequency as depicted in the table. Figure 4 shows the variation of RNF with RCS for a cantilever beam of RCP of 0.05. From the figure, it is clear that the first four relative natural frequencies decrease gradually and significantly. The same trends of variation of natural frequencies are obtained for other relative crack positions of the cantilever beam as mentioned in Table1.

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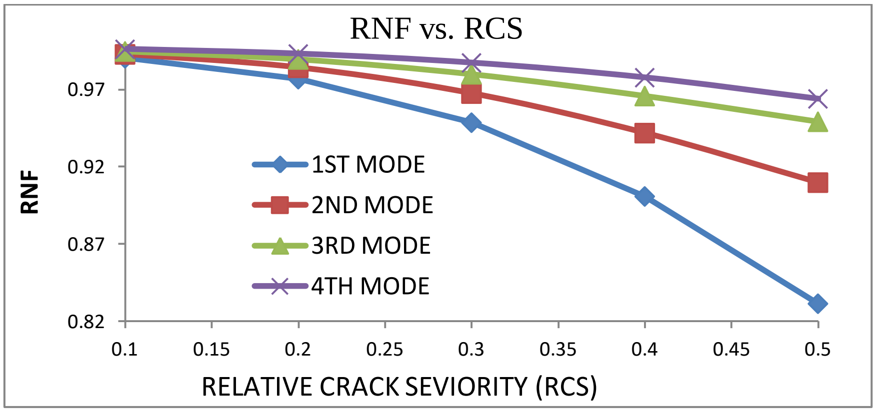

Figure 4: Relation of RCS with RNF

The variations of RNF with the variation of relative crack position were also studied. Figure5 shows the variation of RNF with RCP of a cantilever beam of RCS of 0.1. From the graph, it can be observed that there is a significant abrupt drop of RNF after the RCP of 0.4 i.e. 40% of the total span of the cantilever beam. Then extending the crack position RCP after 0.8 there is an occurrence of slight increase in RNF. This is due to the decrease of total dead weight towards the free end beyond the crack. The same trends of variation of RNF are found as depicted in Table 1.

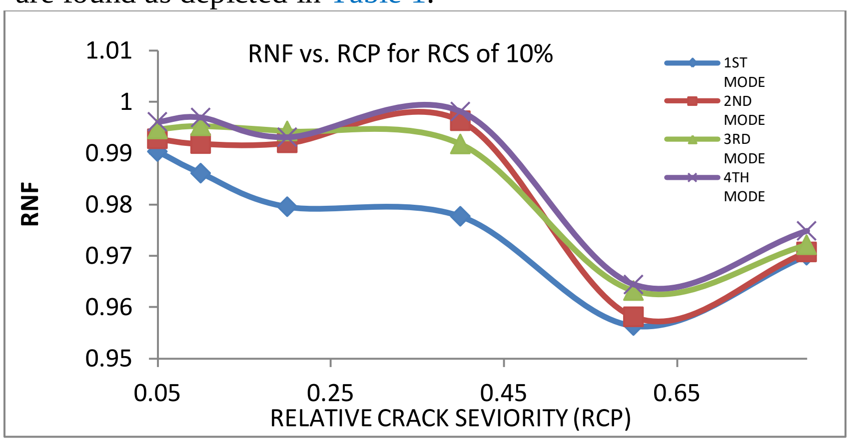

Figure 5: Relation of RCP with RNF

The variations of RNF with first four modes of five cracked samples with RCS of 0.1, 0.2, 0.3, 0.4 and 0.5 for RCP of 0.05, 0.1, 0.2, 0.4, 0.6 and 0.8 were studied. Figure 6 presents the variation of the first four bending RNFs of five models cracked beams with RCP of 0.05. From the figure, it can be observed that the RNF increases with the bending modes and tends to be uniform. However, another finding is that beam of Sample-5 with RCS of 0.5 has a higher deviation of RNF as compared to other beams hence implying an increase in RCS leads to a non-linear variation of RNF.

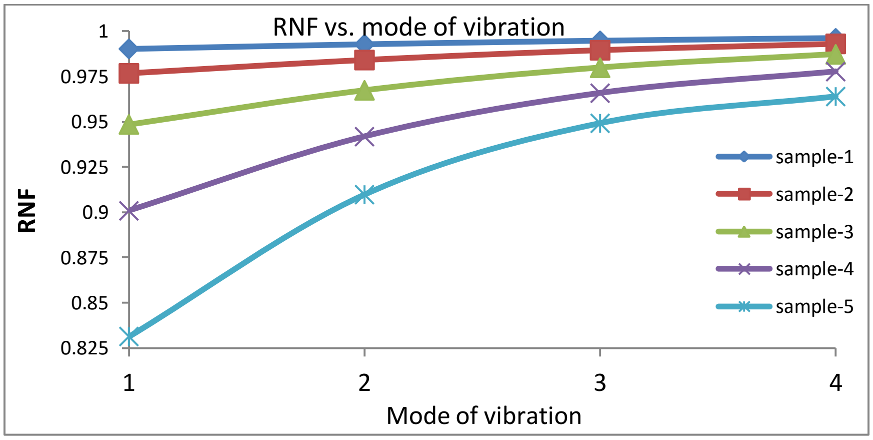

Figure 6:variation of RNF with mode of vibration of cracked beams with RCP 0.05

Figure 7 shows the variation of RNF with bending modes of vibration of six samples of cracked beams with RCP at 0.05, 0.1, 0.2, 0.4, 0.6, and 0.8 for a fixed RCS of 0.1.

Samples 5 and 6 with RCP 0.6 and 0.8 have the lowest RNF exhibiting lower stiffness of the beams.

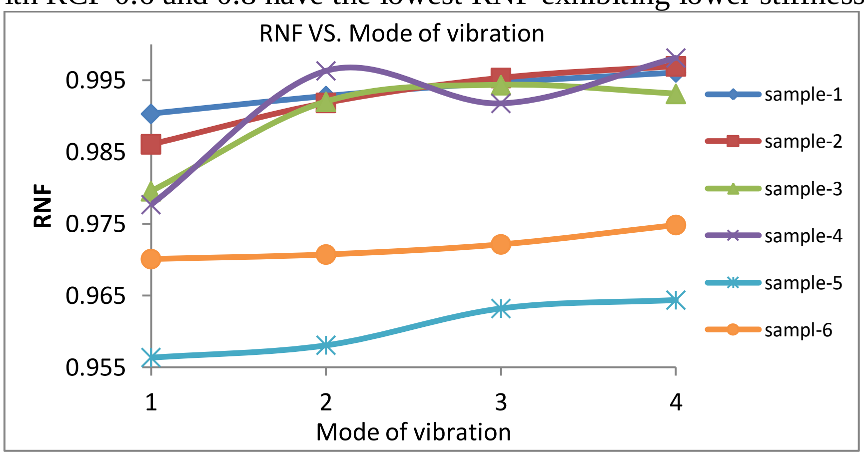

Figure 7: Variation of RNF with mode no. of cracked beams with RCS 0.5

3. Conclusion

Compared to the other natural frequencies, the first four relative natural frequencies have seen alterations that are more notable.

- The rigidity of the beam is decreased when the relative crack severity increases at any place, which significantly reduces the natural frequency.
- RNF suddenly decreases after the RCP of 0.4, or 40% of the whole span.
- There is a tendency for RNF to increase slightly with RCP of 0.8.
- The rise in RCS causes bending modes to vary nonlinearly.

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