



## A COMPUTATIONAL METHODOLOGY FOR SOLVING LINEAR FRACTIONAL PROGRAMMING PROBLEMS

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### ABSTRACT

For dealing with many types of linear inequalities, various methods have been introduced in linear fractional programming method. LFP is a class of optimization problems characterized by the objective function being a ratio of two linear functions, subject to a set of linear constraints. To find the optimality of an objective function. In this paper a computational framework for solving LFP problems efficiently and accurately. The proposed method leverages a reformulation technique that transforms the fractional objective into an equivalent linear programming problem, followed by the application of Cramer's Rule to determine the basic feasible solution. This approach ensures simplicity in implementation and clarity in theoretical interpretation, while maintaining computational effectiveness. Numerical examples are provided to demonstrate the validity and performance of the proposed method in comparison with traditional techniques.

**Keywords:** Linear fractional programming, crammer's rule, linear inequalities, optimization, linear programming, Determinants and optimal solution.

### I. Introduction

Linear Fractional Programming represents the class of mathematical optimization problems in which the objective function is expressed as the ratio of two linear functions, subject to a set of linear constraints. These problems are widely encountered in practical applications, including portfolio optimization, production planning, decision analysis, and resource allocation, where objectives often involve maximizing efficiency or minimizing cost-effectiveness ratios. There are various traditional methods for solving Linear Fractional Problems typically involve transforming the fractional objective into an equivalent linear form using techniques such as the Charnes-Cooper transformation or Dinkelbach's. In this paper, a novel computational framework for solving linear fractional programming problems has been presented by integrating transformation techniques with Cramer's Rule—a classical method from linear algebra for solving systems of linear equations. By utilizing Cramer's Rule, a direct and theoretically elegant way to find basic feasible solutions, enhancing clarity of the solution process is shown.

The primary work in this paper are as follows:

- Firstly, a structured algorithm has been developed that reformulates LFP into an equivalent linear programming problem.
- Then by applying Cramer's Rule and systematically solving the transformed problem, ensuring both accuracy and transparency.
- Finally, effectiveness of the proposed approach is shown through illustrative numerical examples and comparative analysis with existing methods.

Many scientific and engineering applications, which occur in daily life can be ultimately modelled in terms of systems of linear algebraic equations. Hence, crammer's method has been proved better method which can be easily applicable for solving the traditional linear fractional programming problem.



**II. Algorithm**

Step 1: Consider an objective function of two unknown variables with respective constraints

$$Z(x) = \frac{L(x)}{S(x)} = \frac{l_1x_1+l_2x_2+l_0}{s_1x_1+s_2x_2+s_0} \rightarrow \max/\min \quad (2.1)$$

Subject to

$$a_{i1}x_1 + a_{i2}x_2 \leq b_i \quad i = 1,2 \dots m \quad (2.2)$$
$$x_1 \geq 0 \quad x_2 \geq 0$$

Where  $x_1$  and  $x_2$  are non -ve variables.

Step 2: Now convert the given constraints of LFP into equality or standard form

$$Z(x) = \frac{L(x)}{S(x)} = \frac{l_1x_1+l_2x_2+l_0}{s_1x_1+s_2x_2+s_0} \rightarrow \max/\min \quad (2.3)$$

$$a_{i1}x_1 + a_{i2}x_2 = b_i \quad i = 1,2 \dots m \quad (2.4)$$
$$x_1 \geq 0 \quad x_2 \geq 0$$

Step 3: Now consider two simultaneous constraints of the above given problem in  $x_1, x_2, x_3, \dots$  then by Crammer’s rule find the value of the decision variable.

Step 4: After finding the values of decision variable substitute these values in the objective function to find the optimum result.

[Note: For the system of equations  $AX = B$  :

Case1. If  $|A| \neq 0$  then in this case the system of equations is called consistent and has unique solution.

Case2. If  $|A| = 0$  and any of  $|A_1|, |A_2|, |A_3|$  is non zero then in this case the system of equation is inconsistent and will have no solution.

Case3. If  $|A| = 0$  and all of  $|A_1|, |A_2|, |A_3|$  is zero then in this case the system of equation is consistent and will have infinitely many solution. ]

**III. Numerical Example and Results**

Example : Maximize  $Z_1 = \frac{x_1+2x_2+1}{x_1+x_2+1}$

Subject to:  $x_1 + 2x_2 \leq 4$   
 $2x_1 + x_2 \leq 2$   
 $x_1 + x_2 \leq 1$   
 $x_1 \geq 0, x_2 \geq 0$

Solution:

Maximize  $Z_1 = \frac{x_1+2x_2+1}{x_1+x_2+1} \dots\dots\dots(B)$

Subject to:  $x_1 + 2x_2 \leq 4 \dots\dots\dots(1)$   
 $2x_1 + x_2 \leq 2 \dots\dots\dots(2)$   
 $x_1 + x_2 \leq 1 \dots\dots\dots(3)$   
 $x_1 \geq 0, x_2 \geq 0$

Using equation (1) and (2)  $\begin{bmatrix} 1 & 2 & : & 4 \\ 2 & 1 & : & 2 \end{bmatrix}$

$D_{x_1} = \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} = 0$  ;  $x_1 = \frac{D_{x_1}}{D} = 0$

$Z_{max} = 1.5$

$D = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$

$D_{x_2} = \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} = 6$  ;  $x_2 = \frac{D_{x_2}}{D} = 2$

Using equation (1) and (3)  $\begin{bmatrix} 1 & 2 & : & 4 \\ 1 & 1 & : & 1 \end{bmatrix}$

$D_{x_1} = \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} = 2$  ;  $x_1 = \frac{D_{x_1}}{D} = -2$

$D = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -1$

$D_{x_2} = \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} = -3$  ;  $x_2 = \frac{D_{x_2}}{D} = 3$

$Z_{max} = 2.5$ , since the extreme points cannot be negative, therefore the above solution cannot be optimum.



Using equation (2) and (3)  $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} : \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$D_{x_1} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 \quad ; \quad x_1 = \frac{D_{x_1}}{D} = 1$$

$$D = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$$

$$D_{x_2} = \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = 0 \quad ; \quad x_2 = \frac{D_{x_2}}{D} = 0$$

$$Z_{\max} = 1$$

From all the optimum results maximum value is  $Z_{\max} = 1.5$ .

#### IV. Conclusion

In this paper crammer's rule has been used to find the maximum or minimum value of linear fractional programming problem. It has also been observed that after comparison with the results of other method, the obtained results are similar, and found that the proposed method is quite simple and easy to implement, comparative to other algorithms.

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