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THERMO-RADIATION ABSORPTION IN THE PRESENCE OF NON-UNIFORM HEAT SOURCE WITH CONSTANT HEAT AND MASS FLUX NANO FLUID OVER AN EXPONENTIALLY STRETCHING PERMEABLE SHEET WITH VISCOUS DISSIPATION

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Abstract: The present study analyzes the steady boundary layer flow of magneto-nanofluid due to an exponentially permeable stretching sheet with viscous dissipation. In this paper, the effect viscous dissipation on heat transfer and nano particle volume friction is considered. The study reveals that the governing parameters, namely, the magnetic parameter, wall mass suction parameter, Prandtl number, the Lewis number, Eckert number, Brownian motion parameter, and thermo phoresis parameter, have major effects on the flow field, the heat transfer, and the nano particle volume fraction as well as skin friction, local Nusselt number and local Sherwood number has been discussed in detail. Finally, a parametric study was conducted to investigate the influences of various parameters on the fluid flow pattern and heat-transfer performance. The influence of important parameters such as Lewis number , buoyancy ratio parameter ,and also different parameters discussed.

Keywords: Nanoparticle, Heat transfer, viscous dissipation, stretching sheet.

1.Introduction.

A nanofluid is a fluid containing small volumetric quantities of nanometer-sized particles, called nanoparticles. These fluids are envisioned to describe a fluid, in which nanometer-sized particles are suspended,in convectional heat transfer of basic fluids. The nanoparticles used in nanofluids are typically made of metals (Al, Cu), oxides (Al_2O_3, CuO, TiO_2, SiO_2), carbides (SiC), nitrides (AlN, SiN), or nonmetals (graphite, carbon nanotubes). Nanoparticles are particles with a diameter of about 1-100 nm. Nanofluids commonly contain up to a 5% volume fraction of nanoparticles to see effective heat transfer enhancements of the base fluid.To improve the thermal conductivity of these fluids nano/micro-sizedparticle materials are suspended in liquids. Several theoretical and experimental researches have been made to enhance the thermal conductivity of these fluids.Choi [1] was the first person to introduce the word “nanofluids. In nature such flows are encountered in the oceans, lakes, solar ponds, and the atmosphere. They are also responsible for the geophysics of planets. In industry, a familiar example of a binary mixture of fluids is an emulsion, which is the dispersion of one fluid within another fluid. Typical emulsions are oil dispersed within water or water within oil. Another example where the mixture of fluids plays an important role is in multigrade oils. Polymeric-type fluids are added to the base oil so as to enhance the lubrication properties of mineral oil. Moreover, the mixed convection boundary layer problem is also encountered in aerodynamic extrusion of plastic and rubber sheets, cooling of an infinite metallic plate in a cooling path, which may be an electrolyte, crystal growing, the boundary layer along a liquid film in condensation processes, and a polymer sheet or filament extruded continuously from a die or along thread traveling between a feed roll and a windup roll are examples of practical applications of continuous moving surfaces.

Heat transfer phenomena over a stretching sheet have received great attention in chemical and manufacturing processes. The pioneering work of Sakiadis[2], Crane [3] studied a steady flow past a stretching sheet and presented a closed form solution to it.Suction or injection (blowing) of a fluid through the bounding surface can significantly change the flow field. Recently the effects of variable suction and thermophoresis on steady MHD flow over a permeable inclined plate was analyzed by Alam et al. [4] while the heat and mass transfer of thermophoretic hydromagnetic flow with lateral mass flux, heat source.Liu [5] investigated the flow and heat transfer of an electrically conducting fluid of second grade over a stretching sheet subject to a transverse magnetic field. Koo and Kleinstreuer [6] have investigated the effects of viscous dissipation on the temperature field using dimensional analysis and experimentally validated computer simulations. Yazdi et al. [7] evaluates the slip MHD flow and heat transfer of an electrically conducting liquid over a permeable surface in the presence of the viscous dissipation effects under convective boundary conditions.[8-11].Evaluates the slip MHD flow and heat transfer of an electrically conducting liquid over a permeable surface in the presence of the viscous dissipation effects under convective boundary conditions[12]. Alam et al. [13] presented an analysis of the Soret and Dufour effects on free convective heat and mass transfer flow in a porous medium with time-dependent temperature and concentration. Beg et al. [14] investigated numerically the free convection magnetohydrodynamic heat and mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects. Various other aspects dealing with the Soret and Dufour effects on steady boundary layer flow with combined heat and mass transfer problems have been reported (Afify [15], Chamkha [16,17], Tsai and Huang [18,19], Seddeek [20], Abdallah [21], Alam and Rahman[22], Ferdows et al [23], El-Aziz [24,25]).

2. Mathematical formulation

Consider the steady boundary layer flow of nanofluid over an exponentially stretching sheet in presence of a transverse magnetic field. The governing equations of motion and the energy equation may be written in usual notation as

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = 0 \tag{2.1}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \nu\,\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho f}\,u \tag{2.2}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial y^2} + \frac{(\rho c)_p}{(\rho c)_f}\left[D_B\frac{\partial N}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu_f}{(\rho c)_f}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

$$u\,\frac{\partial N}{\partial x} + v\,\frac{\partial N}{\partial y} = D_B\frac{\partial^2 N}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{2.4}$$

The boundary conditions are

$$u = U_w,\, v = v_w,\, T = T_w = T_\infty + T_0 e^{\frac{x}{2L}},\, N = N_w = = N_\infty + N_0 e^{\frac{x}{2L}} \quad \text{at } y = 0$$

$$u \to 0, T \to T_\infty, N \to N_\infty \quad as\; y \to \infty \tag{2.5}$$

Here the variable magnetic field *B(x)* is taken the form

$B(x) = B_0 e^{\frac{x}{2L}}$ where *B0* is the constant (2.6)

The stretching velocity U_w is given by

$$U_w(x) = c\, e^{\frac{x}{L}} \tag{2.7}$$

Where $c > 0$is stretching constant.
Here $v_w$is the variable wall mass transfer velocity and is given by

$$v_w(x) = v_0 e^{\frac{x}{2L}} \quad (2.8)$$

Where$v_0$is a constant with $v_0 < 0$ formass suction and$v_0 > 0$ for mass injection

The continuity equation (2.1) is satisfied by the Cauchy Riemann equations
$u = \frac{\partial \psi}{\partial y}$and $v = \frac{\partial \psi}{\partial x}$ where $\psi(x,y)$is the stream function. (2.9)

In order to transform the equations (2.2) to (2.5) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced

$$\psi = \sqrt{2vLc}\; f(\eta) e^{\frac{x}{2L}},\; \eta = y\sqrt{\frac{c}{2vL} e^{\frac{x}{2L}}}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\; \beta(\eta) = \frac{N - N_\infty}{N_w - N_\infty} \quad (2.10)$$

$$P_r = \frac{v}{\alpha}, \quad Le = \frac{v}{D_B}, M = \frac{2L\sigma B_0^2}{\rho c}, \quad E_c = \frac{U_w^2}{c_f\,(T_w - T_\infty)}$$

$$N_b = \frac{D_B(\rho c)_p (N_w - N_\infty)}{v(\rho c)_f}, \quad N_t = \frac{D_T(\rho c)_p (T_w - T_\infty)}{T_\infty(\rho c)_f v}$$

In view of the equation (2.7), the equations (2.2) to (2.6) transform into

$$f''' + f f'' - 2 f'^2 - M f' = 0 \quad (2.11)$$

$$\theta'' + P_r(f\theta' - f'\theta) + Nb\theta'\beta' + Nt\theta'^2 + Ecf''^2 = 0 \quad (2.12)$$

$$\beta'' + Le\,(f\beta' - f'\beta) + \frac{N_t}{N_b}\theta'' = 0 \quad (2.13)$$

The transformed boundary conditions can be written as

$$f = S, f' = 1, \theta = 1, \beta = 1 \quad at\ \eta = 0$$

$$f' \to 0, \theta \to 0, \beta \to 0 \;\; as\ \eta \to \infty \quad (2.14)$$

Physical quantities of interest are Local skin friction coefficientC_f, Local Nusselt number $N_u$and

Local Sherwood number S_h, defined as

$$C_f = \frac{v}{U_w^2 e^{\frac{2x}{L}}} \left(\frac{\partial u}{\partial y}\right)_{y=0},\; N_u = \frac{-x}{(T_w - T_\infty)} \left(\frac{\partial T}{\partial y}\right)_{y=0}, S_h = \frac{-x}{(N_w - N_\infty)} \left(\frac{\partial N}{\partial y}\right)_{y=0} \quad (2.15)$$

or by introducing the transformations (2.10), we have

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$$\sqrt{2Re_x}C_f = f''(0), \quad \frac{N_u}{\sqrt{2Re_x}} = -\sqrt{\frac{x}{2L}}\theta'(0), \quad \frac{S_h}{\sqrt{Re_x}} = -\sqrt{\frac{x}{2L}}\beta'(0) \qquad (2.16)$$

Where $Re_x = \frac{U_w x}{v}$ is the local Reynolds Number

3. **Results and discussion**

The velocity of the fluid decreases with increases in the magnetic field parameter due to an increase in the Lorentz drag force that opposes the fluid motion is observed in Fig.1. The effect of thermophoresis parameter (*Nt*) on temperature is shown in Fig. 2. An increase in *Nt*, the temperature of the fluid increases. The effect of Brownian motion parameter (*Nb)* on the temperature is plotted in Fig.3. The figure reveals that the temperature of the fluid increases with increasing values *Nb* . From Figure. 4 it is seen that the skin friction increases with an increase *M* or S. The variations of *Nt*, *Le* and *S* on reduced Nusselt number is shown in Figure.5. It is observed that the reduced Nusselt number increases with an increase the mass flux parameter and decrease with an increasing the parameters *Nt or Le*. The effect of *Nt*, *Le* and *S* on nanofluid Sherwood number is shown in Fig.6.

The data in Table.1 indicate how the reduced Nusselt and Sherwood numbers are affected by the changes in the Prandtl number Pr, the Brownian motion parameter Nb and the buoyancy parameter Nb when the rest of the parameters are fixed at the values indicated in the table heading. For every combination of Nb and Nr, both the reduced Nusselt and reduced Sherwood numbers increase with the increase in Pr. For a fixed Pr, both the reduced Nusselt number and the reduced Sherwood number decrease as Nb and Nr each increase.

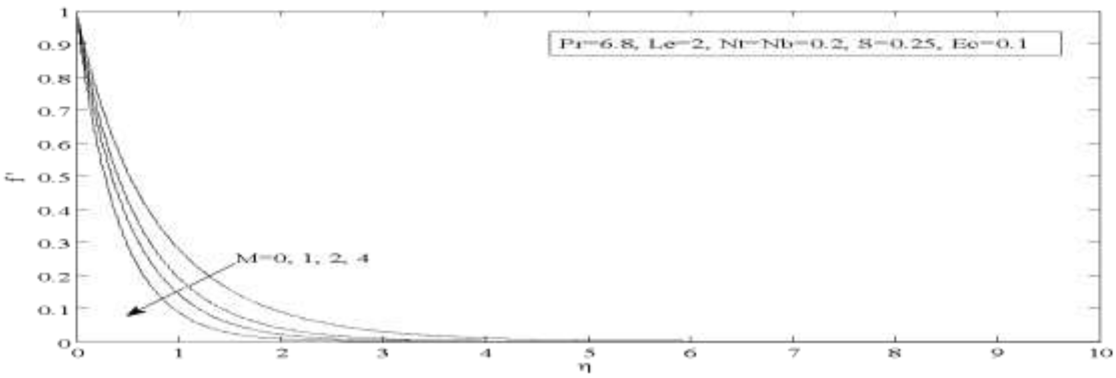

Figure 1 Velocity for different *M*

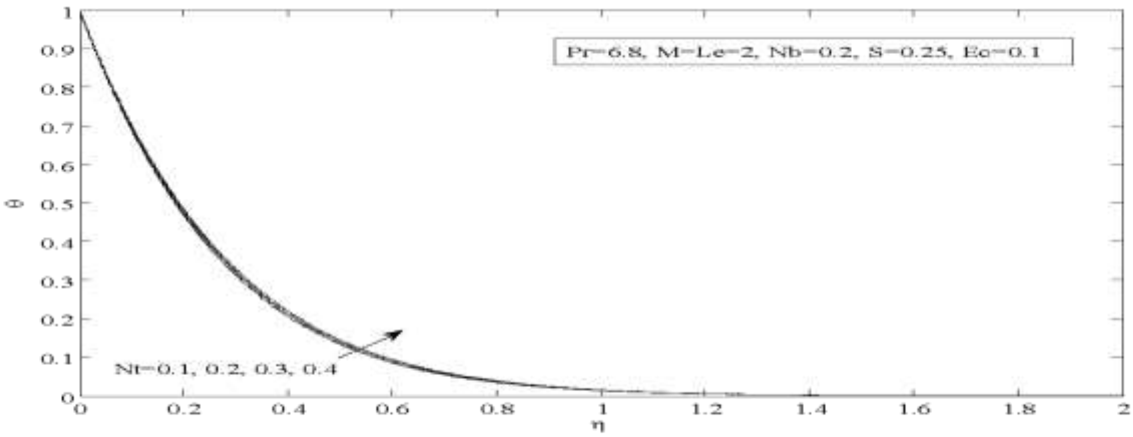

Figure 2 Temperature for different *Nt*

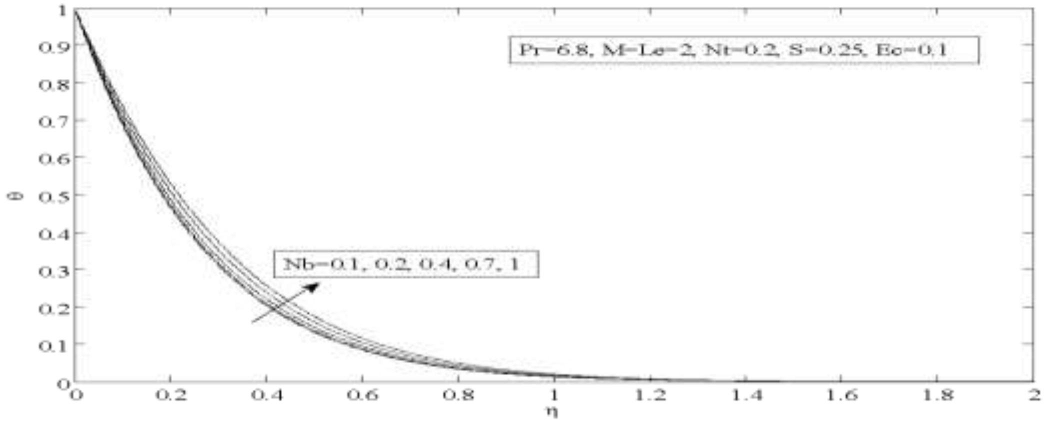

Figure 3 Temperature for different *Nb*

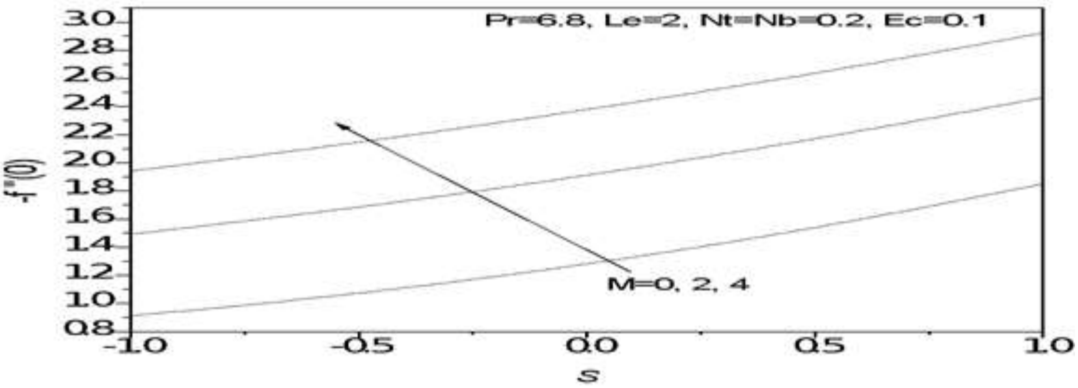

Figure 4 Effect of *S* and *M* on the reduced skin friction

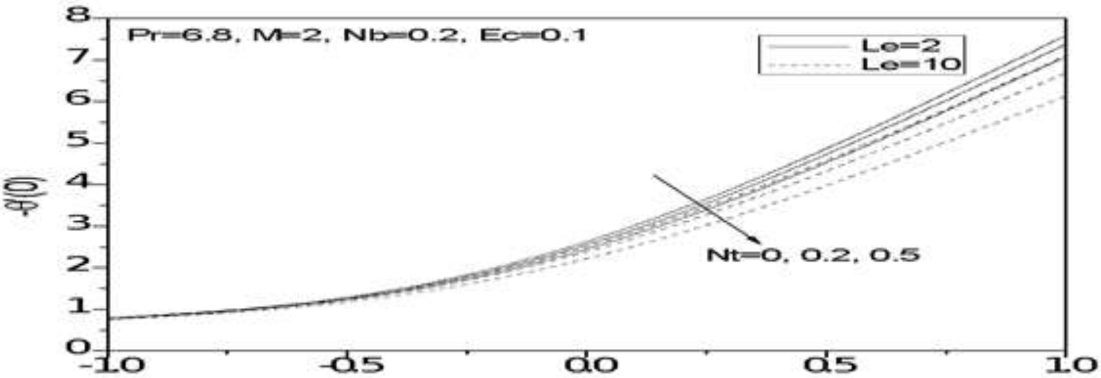

Figure 5 Effect of *Nt*, *Le* and *S* on the reduced Nusselt number

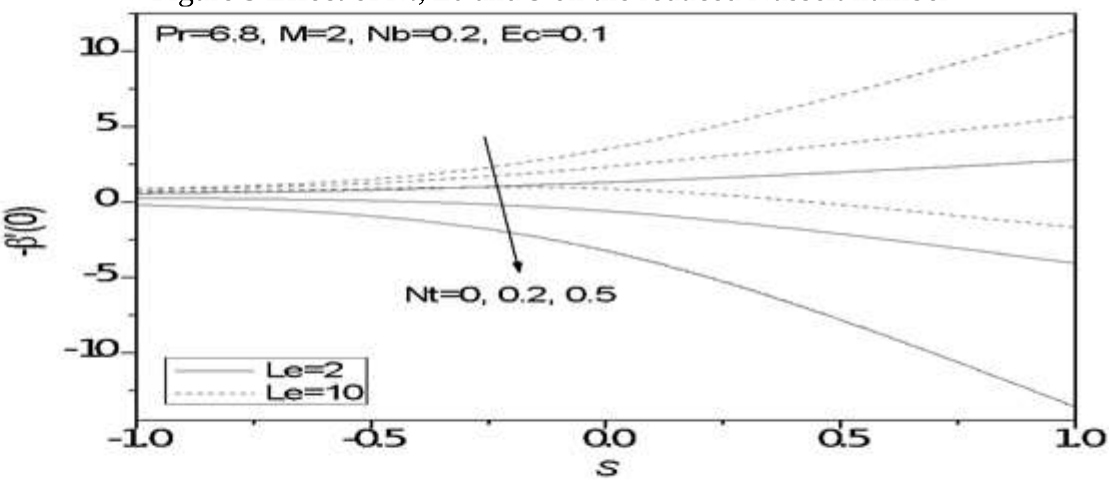

Figure 6 Effect of *Nt*, *Le* and *S* on the reduced Sherwood number

Table.1 Variation of Nur and Shr with Pr,Nb and Nr for Nt=0.1, Nc=10 and Le = 10

Nb	Nr	Pr=1			Pr=5	Pr=10	
		Nur	Shr	Nur	Shr	Nur	Shr
0.1	0.1	0.5843	0.9965	0.3283	1.2239	0.48453	1.3817
	0.2	0.5593	0.9953	0.3258	1.2236	0.48103	1.3587
	0.3	0.5343	0.9941	0.3233	1.2233	0.47753	1.3357
	0.4	0.5093	0.9929	0.3208	1.223	0.47403	1.3127
	0.5	0.4843	0.9917	0.3183	1.2227	0.47053	1.2897
	0.6	0.4593	0.9905	0.3158	1.2224	0.46703	1.2667
	0.7	0.4343	0.9893	0.3133	1.2221	0.46353	1.2437
	0.8	0.4093	0.9881	0.3108	1.2218	0.46003	1.2207
0.3	0.1	0.3843	1.1839	0.3083	1.2215	0.45653	1.1977
	0.2	0.3593	1.1719	0.3058	1.2212	0.45303	1.1747
	0.3	0.3605	1.1599	0.3033	1.2209	0.44953	1.1517
	0.4	0.3617	1.1479	0.3008	1.2206	0.44603	1.1287
	0.5	0.3629	1.1359	0.2983	1.2203	0.44253	1.1057
	0.6	0.3641	1.1239	0.2958	1.22	0.43903	1.0827
	0.7	0.3653	1.1119	0.2933	1.2197	0.43553	1.0597
	0.8	0.3665	1.0999	0.2908	1.2194	0.43203	1.0367
0.5	0.1	0.3677	1.1935	0.2883	1.2191	0.42853	1.0137
	0.2	0.3689	1.1785	0.2858	1.2188	0.42503	0.9907
	0.3	0.3701	1.1623	0.2833	1.2185	0.42153	0.9677
	0.4	0.3713	1.1483	0.2808	1.2182	0.41803	0.9447
	0.5	0.3725	1.1235	0.2783	1.2179	0.41453	0.9217
	0.6	0.3737	1.0435	0.2758	1.2176	0.41103	0.8987
	0.7	0.3749	0.8935	0.2733	1.2173	0.40753	0.8757
	0.8	0.3761	0.7435	0.2708	1.217	0.40403	0.8527

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