



## **A NUMERICAL INVESTIGATION OF MHD MICRO POLAR FLUID OVER A STRETCHING PERMEABLE SHEET WITH HEAT GENERATION AND THERMAL SLIP FLOW**

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**Abstract:** The two-dimensional boundary layer of an electrically conducting magnetohydrodynamic micropolar fluid spreading over a permeable stretching sheet with viscous dissipation in the presence of heat generation and temperature-dependent slip flow is attempted to be numerically solved in this study. The governing equations have been converted into a system of ordinary differential equations by using the similarity transformations. Due to their extreme nonlinearity, these differential equations are impossible to solve analytically. As a result, it has been solved using the bvp4c MATLAB solver. For varying values of the governing parameters, namely the material parameter, magnetic parameter, thermal slip parameter, radiation parameter, Prandtl number, and Eckert number, numerical results are obtained for the skin-friction coefficient, the couple wall stress, and the local Nusselt number in addition to the velocity, microrotation, and temperature profiles. For the heat generation parameter, the temperature profile rises.

**Keywords:** Heat creation, heat transfer, thermal slip flow, radiation, viscous dissipation, and micropolar fluid.

### **1. Introduction**

Eringen was the first to introduce and formulate the fundamental continuous theory for micropolar fluids [1-2]. This theory has been used to explore different flow situations, such as the flow of low concentration suspensions, liquid crystals, blood, colloidal fluids, and ferro-liquids, for which the classical Navier-Stokes theory is unsuitable. It achieves this by take into account the microscopic effects that result from the local structure and micro motions of the fluid elements.

Applications in engineering and geography, including geothermal reservoirs, thermal insulation, nuclear reactor cooling, and enhanced oil recovery, are interested in the heat and mass transmission from various geometries imbedded in a porous media, which is known as magnetohydrodynamics (MHD). In various engineering processes, including metallurgical and polymer extrusion processes, a molten liquid is cooled while simultaneously being drawn into a cooling system. The cooling fluid and the strain rate have a significant influence on the fluid mechanical properties of the final product. Due to the possibility of controlling their flow by external magnetic fields to improve the quality of the final product, various polymer fluids with outstanding electromagnetic properties – such as polyethylene oxide and polyisobutylene solution in Cetane – are mainly used as cooling fluids. The boundary layer flow investigated by Sakiadis ([3-4]) is caused by a moving plate in a stationary ambient fluid. Since then, other authors have investigated various aspects of the topic, including Fang [5], Fang and Lee [6], and White [7]. The impact of a magnetic field on natural convection flow on a vertical surface has been studied by Chamkha and Khaled [8]. The effects of joule heating on a boundary layer of an MHD micropolar fluid over a non-isothermal permeable stretching sheet with variable electric conductivity were examined by Majidiana et al. [9]. They found that while the thickness of the temperature profile increases as the magnetic parameter coefficient increases, the thickness of the velocity boundary layer decreases. The heat transfer for MHD micropolar fluids flowing through a porous medium over a stretching surface in the presence of a



magnetic field and heat source was examined by Ahmad and Hussain [10]. Their findings indicated that as the values of the micropolar parameters increased, the microrotation increased slightly as one moved away from the boundary. Mishra et al. [11] studied the effect of a transverse magnetic field on a doubly stratified micropolar fluid, analyzing the planar flow of an electrically conducting incompressible viscous fluid on a vertical plate with variable wall temperature and concentration. Their findings indicated that the skin friction coefficient increases with the magnetic parameter  $M$  and the couple number  $N$ . The work of Mahmood and Waheed [12] showed the MHD flow and heat transfer of a micropolar fluid over a stretching surface, incorporating heat generation (or absorption) and slip velocity. Chaudhary and Abhay [13] examined how chemical reactions affect MHD micro-polar fluid flow over a vertical plate in the slip-flow regime.

In industrial applications, the influence of radiation on heat transfer problems and unsteady free convection flow has become increasingly important. At elevated working temperatures, the effects of radiation can be extreme. Many engineering processes operate at elevated temperatures, making it essential to comprehend radiation heat transfer for creating reliable machinery, nuclear power plants, gas turbines, and other propulsion systems, in addition to designing aircraft, missiles, satellites, and spacecraft. Sharma et al. [14] investigated how chemical reactions influence the flow of magneto-micropolar fluid from a radiative surface with variable permeability. Ibrahim et al. [15] studied the case of mixed convection flow of a micropolar fluid through a semi-infinite, continuously moving porous plate with a suction velocity that changes normal to the plate, considering the effects of viscous dissipation and thermal radiation. In a saturated porous medium, Oahimire and Olajuwon [16] studied how mass and heat transfer influence the unsteady flow of a micro-polar fluid over an infinite moving permeable plate, considering factors such as radiation absorption, thermo-diffusion, and a transverse magnetic field. Modather et al. [17] studied the MHD heat and mass transfer flow of a micro-polar fluid over a vertical permeable plate in porous media, excluding the influences of thermal radiation, heat generation, and thermodiffusion. Abbasi et al. [18] introduced the study of MHD two-dimensional boundary layer flow of Jeffrey nanofluid over a stretching sheet with thermal radiation. Hussain et al. [19] examined the flow problem that occurs when a surface with convective conditions is stretched in a third-grade MHD nanofluid with heat radiation involved. Hayat et al. [20] studied the influence of thermophoresis and Brownian motion on the two-dimensional boundary layer flow of an Oldroyd-B nanofluid, considering heat generation and thermal radiation.

The viscous incompressible electrically conducting micropolar fluid over a stretching permeable sheet with heat generation and the thermal slip effect is examined in this work. The governing equations were converted into a set of ordinary differential equations using similarity transformations. Since these equations are nonlinear and cannot be solved analytically, the `bvp4c` MATLAB solver was utilized to solve them. Results for temperature, velocity, and microrotation functions are obtained for a variety of significant parameters, including material, magnetic, Eckert number, and first- and second-order slip velocity parameters. Additionally, the rate of heat transfer, skin friction, and pair wall tension have been calculated.

## 2. MATHEMATICAL FORMULATION

Consider a radiative micropolar fluid with an incompressible laminar two-dimensional MHD moving across a permeable planar surface. The  $y$ -axis is normal to the plate, and the flow is considered to be in the  $x$ -direction, which is taken along the plate in a forward direction. In the  $y$ -direction, which is normal to the direction of flow, a variable magnetic field is applied. There is no application of an external electric field. Furthermore, the magnetic Reynolds number is so low that, in comparison to the external magnetic field, the magnetic field created by the moving fluid is insignificant. It is believed that the electrical conductivity takes the following form:

$$\sigma = \sigma_0 \mu \quad (2.1)$$

Where  $\sigma_0$  is a constant

For the flow under study, it is relevant to assume that the applied magnetic field strength has the form [36]:

$$B(x) = B_0 x^{-1/2} \quad (2.2)$$

Where  $B_0$  is a constant

Under standard boundary conditions, the governing equations for momentum, temperature, and angular velocity fields within the boundary layer are expressed as follows: Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.3)$$

Linear momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \nu + \frac{\kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 B_0^2}{\rho x} u^2 + \frac{\kappa}{\rho} \frac{\partial N}{\partial y} \quad (2.4)$$

Angular momentum equation

$$\rho j \left( u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \gamma \frac{\partial^2 N}{\partial y^2} - \kappa \left( 2N + \frac{\partial u}{\partial y} \right) \quad (2.5)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \left( \frac{\mu + \kappa}{\rho c_p} \right) \left( \frac{\partial u}{\partial y} \right)^2 - \frac{\partial q_r}{\partial y} + \frac{\sigma B_0^2}{\rho c_p x} u^3 \quad (2.6)$$

The boundary conditions for the velocity, Angular Velocity and temperature fields are

$$u = U_w = Cx, v = v_w, N = -s \frac{\partial N}{\partial y}, T = T_w + D_1 \frac{\partial T}{\partial y} \quad \text{at } y = 0$$

$$u \rightarrow 0, N \rightarrow 0, T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty \quad (2.7)$$

In this context,  $u$  and  $v$  denote the velocity components along the  $x$  and  $y$  axes, respectively. The other variables are defined as follows:  $\nu$  represents kinematic viscosity;  $N$  is angular velocity;  $\kappa$  denotes vortex viscosity;  $\rho$  indicates fluid density;  $\gamma$  refers to spin gradient viscosity;  $j$  signifies micro inertia per unit mass;  $B(x)$  and  $\sigma$  represent variable magnetic field and electrical conductivity, respectively;  $T$  is temperature;  $k$  indicates thermal conductivity;  $q_r$  denotes radiative heat flux;  $c_p$  signifies specific heat at constant pressure;  $C$  is a positive constant; and  $s$  is the microrotation parameter. When the microrotation parameter  $s$  is set to 0, we get  $N(x, 0) = 0$ . This indicates a no-spin condition, meaning that microelements in a concentrated particle flow near the wall cannot rotate. When  $s = 0.5$ , the case leads to the antisymmetric component of the stress tensor disappearing, representing weak concentrations. The situation that corresponds to  $s = 1.0$  exemplifies turbulent boundary layer flows.

By using the Rosseland approximation the radiative heat flux  $q_r$  is given by

$$q_r = -\frac{4\sigma_1}{3k^*} \frac{\partial T^4}{\partial y} \quad (2.8)$$

where

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (2.9)$$

In view of equations (2.8) and (2.9), eqn. (2.6) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left( \frac{k}{\rho c_p} + \frac{16\sigma_1 T_\infty^3}{3k^*} \right) \frac{\partial^2 T}{\partial y^2} + \left( \frac{\mu + \kappa}{\rho c_p} \right) \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho c_p x} u^3 \quad (2.10)$$

The continuity equation (2.3) is satisfied by equations



$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \quad (2.11)$$

where  $\psi(x, y)$  is the stream function.

In order to obtain local similarity solution of the problem, following transformations are introduced:

$$\eta = \sqrt{C/\nu} y, \psi = \sqrt{C\nu} x f(\eta), N = C\sqrt{C/\nu} x h(\eta) \quad (2.12)$$

$$T = T_\infty + \frac{D}{\kappa} \left( \frac{\nu}{C} \right) \left( \frac{x}{L} \right)^2 \theta(\eta), K = \frac{\gamma}{\mu j}, \lambda = \frac{\kappa}{\mu}, B = \frac{\nu}{Cj}$$

$$M = 1 + \frac{\sigma_0 B_0^2}{\rho}, Pr = \frac{\rho \nu c_p}{k}, Ec = \frac{k}{D} \frac{L^2 C^3}{\sqrt{C\nu} c_p}, R = \frac{16\sigma_1 T_\infty^3 \rho c_p}{3k * k}$$

where  $f(\eta)$  is the dimensionless stream function,  $\theta$  is the dimensionless temperature,  $\eta$  is the similarity variable,  $C$  and  $D$  are equation constants,  $M$  is the magnetic parameter,  $Ec$  is the Eckert number,  $\lambda$  is the vertex viscosity parameter,  $L$  is the characteristic length,  $K$  and  $B$  are the dimensionless material parameters,  $R$  is the radiation,  $Pr$  is the Prandtl number.

In view of equations (2.11) and

(2.12), the equations (2.4), (2.5) and (2.10) transform into

$$(1 + \lambda) g''' + g g'' + K h' - g'^2 - M g' = 0 \quad (2.13)$$

$$K h'' + g h' - g' h - \lambda B (2h + g'') = 0 \quad (2.14)$$

$$\left( 1 + \frac{4}{3} R d \right) \frac{1}{Pr} \theta'' + g \theta' - 2 g' \theta + Ec (1 + \lambda) (g'')^2 + Ec (M) g'^3 + Q \theta = 0 \quad (2.15)$$

The corresponding boundary conditions are

$$g(0) = f_w, g'(0) = 1, h(0) = 0, \theta(0) = 1 + \beta \theta'(0)$$

$$g' = h = \theta = 0 \quad \text{as} \quad \eta \rightarrow \infty \quad (2.16)$$

where the primes denote differentiation with respect to  $\eta$  and the  $\beta = D_1 \sqrt{C/\nu}$  is the thermal slip parameter

The physical quantities of interest are the skin friction coefficient  $C_{fx}$ , the local couple wall stress  $M_{wx}$  and the local Nusselt number  $Nu_x$  which are defined as

$$C_{fx} = \frac{2}{\rho U_w^2} \left[ (\mu + \kappa) \left( \frac{\partial u}{\partial y} \right)_{y=0} + \kappa (N)_{y=0} \right] = 2(1 + K) Re_x^{-1/2} g''(0) \quad (2.17)$$

$$M_{wx} = \gamma \left( \frac{\partial N}{\partial y} \right)_{y=0} = \frac{\gamma C^2}{\nu} x h'(0) \quad (2.18)$$

$$Nu_x = -\frac{x}{T_w - T_\infty} \left( \frac{\partial T}{\partial y} \right)_{y=0} = -Re_x^2 \theta'(0) \quad (2.19)$$

Our main aim is to investigate how the values of  $g''(0)$ ,  $h'(0)$  and  $-\theta'(0)$  vary in terms of the various parameters.

### 3 SOLUTION OF THE PROBLEM

Equations (2.13) to (2.15) were simplified to a system of first - order differential equations and solved using a MATLAB boundary-value problem solver called bvp4c. The program handles boundary-value problems for ordinary differential equations structured as  $y' = f(x, y, p)$ ,  $a \leq x \leq b$ , using a collocation method that considers general nonlinear two-point boundary conditions  $g(y(a), y(b), p)$ . Here,  $p$  represents a vector containing unknown parameters. Boundary value problems (BVPs) occur in a



variety of forms. Virtually any BVP can be configured to the solution using bvp4c. First, the differential equations must be expressed as a system of first- order ordinary differential equations.

#### 4 RESULTS AND DISCUSSION

As outlined in section 3, the governing equations (2.13) - (2.15), along with the boundary conditions (2.16), are integrated. To gain a clear understanding of the physical problem, the velocity, angular velocity, and temperature have been examined by assigning numerical values to the parameters involved in the problem.

Figures 1a, 1b, and 1c illustrate the influence of the viscosity parameter ( $\lambda$ ) on velocity, angular velocity, and temperature, respectively. With increasing  $\lambda$ , the fluid's velocity increases (see Fig. 1a). The angular velocity rises from the sheet and diminishes away from it (see fig.1b). As the viscosity parameter exerts its influence, the temperature of the fluid rises. The influence of the magnetic parameter ( $M$ ) on velocity, angular velocity, and temperature profiles is illustrated in Figs. 2a, 2b, and 2c, respectively. As the magnetic parameter increases, the fluid's velocity is observed to decrease (refer to fig.2a). It has been discovered that the magnetic parameter slows down the velocity at every location within the flow field. This is due to the fact that applying a transverse magnetic field generates a resistive force (the Lorentz force) akin to drag, which opposes the fluid flow and thereby decreases its speed. As depicted in fig. 2b, the angular velocity of the fluid diminishes close to the sheet and rises at a distance from it, with the magnetic parameters affecting this behavior. As the magnetic parameter is raised, the temperature of the fluid increases (see fig. 2c).

The impacts of the material parameters on velocity, angular velocity, and temperature are illustrated in Figs. 3a, 3b, and 3c, respectively. The impact of the material parameter leads to an increase in velocity and angular velocity (fig. 3a & 3b). As the material parameter (3c) increases, the fluid's temperature decreases. Figs. 4a, 4b, and 4c illustrate the impact of suction/injection on velocity, angular velocity, and temperature, respectively. It is observed that the velocity, angular velocity, and temperature decline as the suction/injection parameter increases. Fig. 5 illustrates how the thermal slip parameter affects temperature. The fluid's temperature is noted to rise as  $\beta$  increases. Figure 6 illustrates how the temperature profile is influenced by the Eckert number. It is observed that temperature rises as  $Ec$  increases. Fig. 7 shows the influence of radiation on temperature. It is noted that the fluid's temperature rises as  $R$  increases. Figure 8 illustrates how temperature is influenced by the Prandtl number. The fluid's temperature is observed to decrease as  $Pr$  increases. As illustrated in Fig. 9, the fluid temperature rises as the heat generation parameter increases.

As indicated in Table 1, skin friction decreases as  $\lambda$  and  $K$  values rise, while it grows for  $M$  and  $fw$ . With increasing values of  $\lambda$ ,  $M$  and  $fw$ , the couple wall stress decreases, while it increases for  $K$ . For increasing values of  $M$ , the Nusselt number decreases, while it increases for  $K$ ,  $\lambda$ , and  $fw$ .

#### 5 CONCLUSIONS

This study examines the unsteady mixed convection flow of a viscous, incompressible, electrically conducting micropolar fluid on a vertical, impermeable stretching surface, considering radiation and thermal slip effects. By means of a similarity transformation, the governing equations are approximated as a system of non-linear ordinary differential equations. Various values of the dimensionless parameters of the problem are used to perform numerical calculations. It has been discovered that

1. The velocity and angular velocity decrease as the temperature increases with a rise in the magnetic parameter.
2. The velocity and temperature rise as the viscosity parameter increases.
3. The fluid's angular velocity rises in proximity to the sheet and diminishes at a distance from it as the viscosity parameter increases.
4. The fluid's temperature rises as the Eckert number and thermal radiation parameters increase.

5. The skin friction decreases the suction/injection or material parameter (K).
6. The couple wall stress rises as the material parameter (K) increases, but it diminishes with a higher viscosity parameter.
7. The local Nusselt number decreases under the influence of the thermal radiation parameter , while it increases with increasing material parameter (K) .
8. The temperature profile rises for the heat generation parameters.

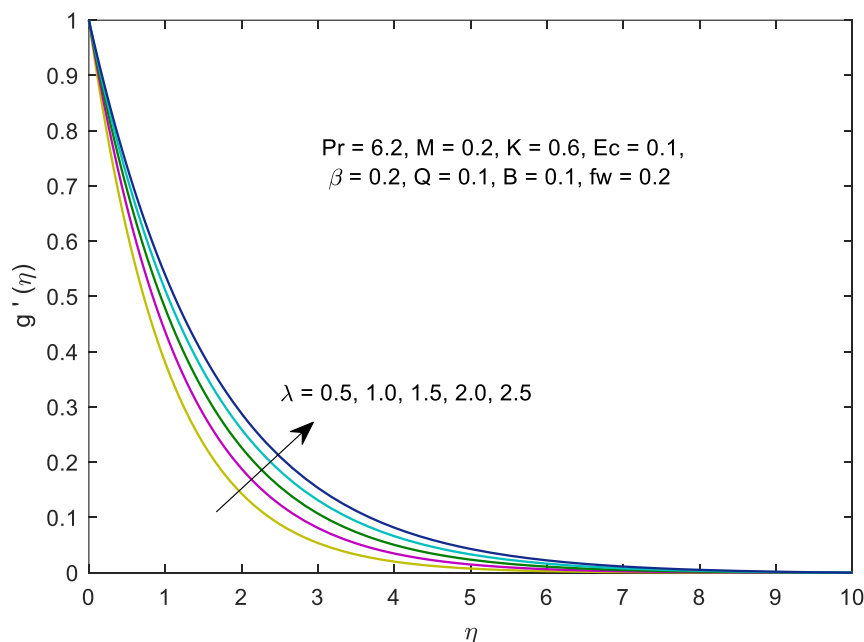


Fig.1a Velocity for different values of  $\lambda$

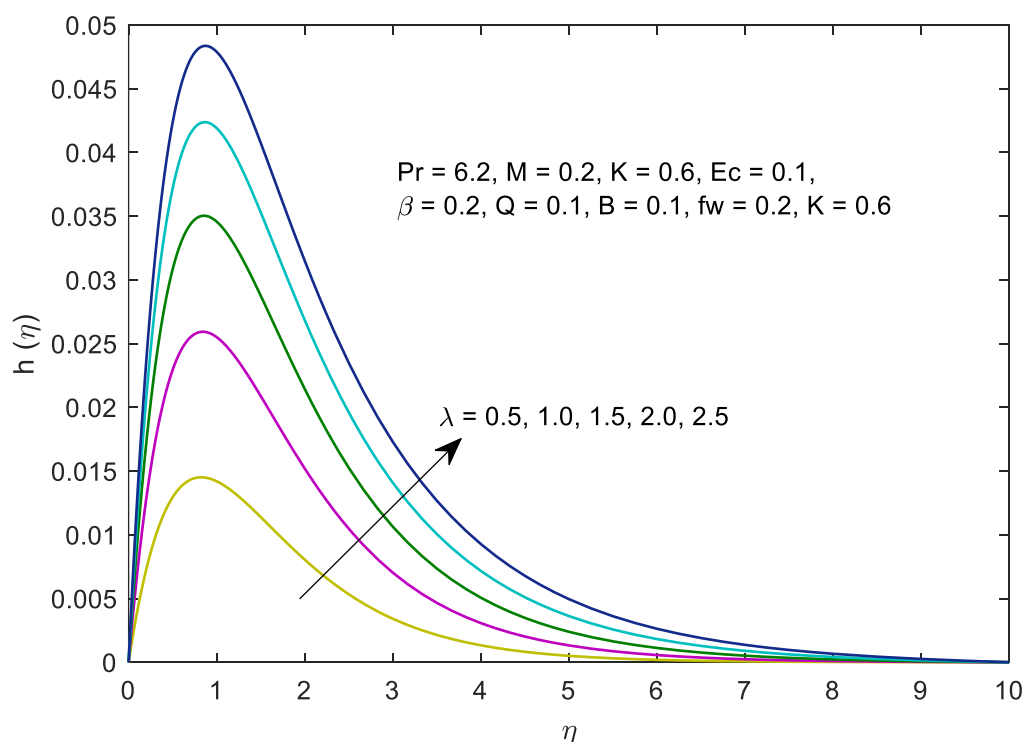


Fig.1b Angular velocity for different values of  $\lambda$

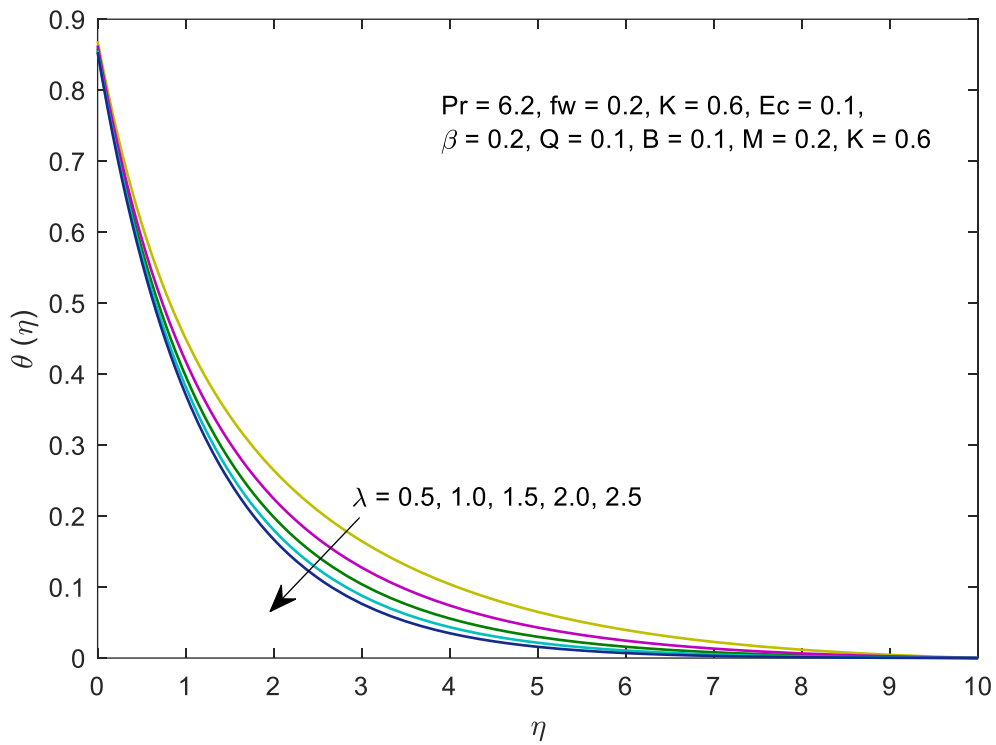


Fig.1c Temperature for different values of  $\lambda$

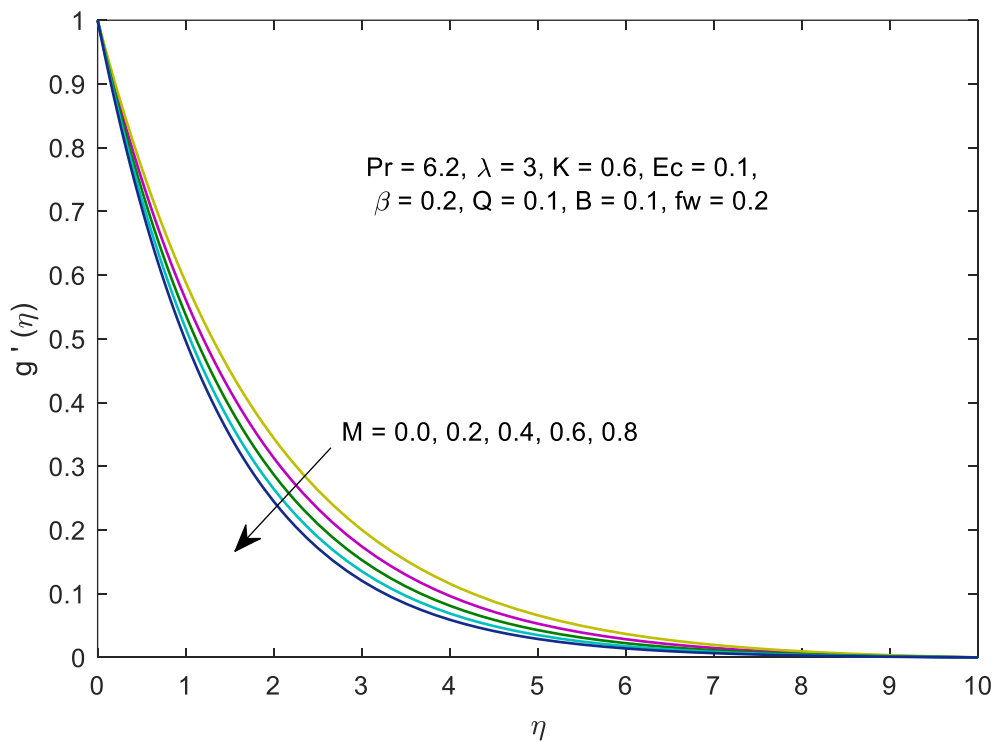


Fig.2a Velocity for different values of  $M$

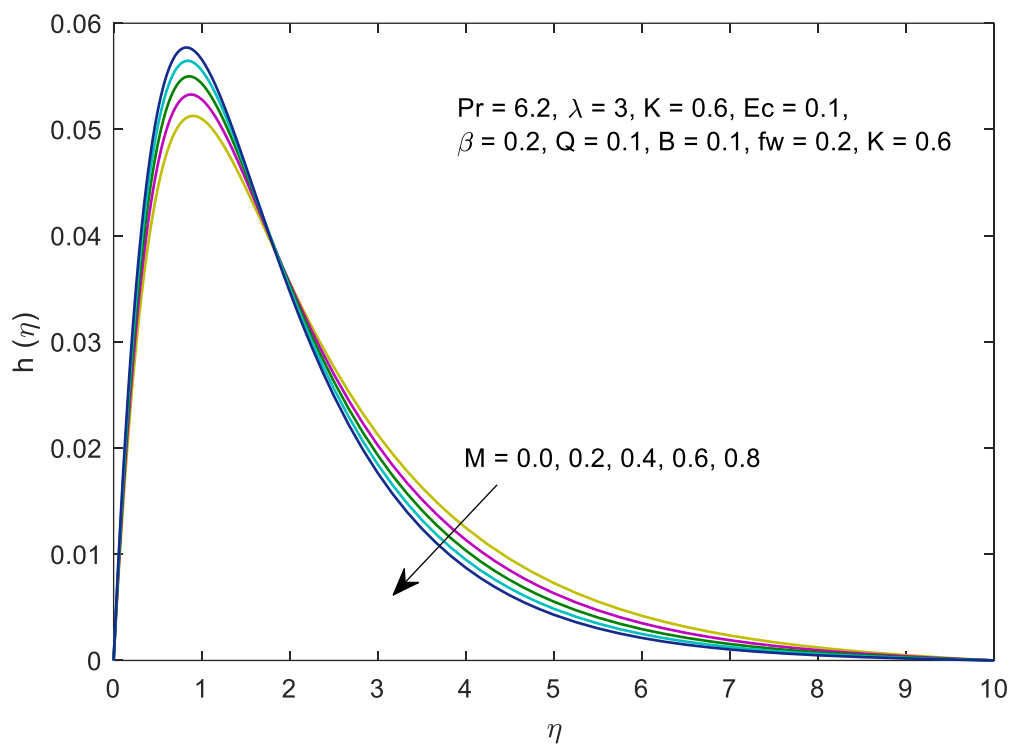


Fig.2b Angular velocity for different values of  $M$

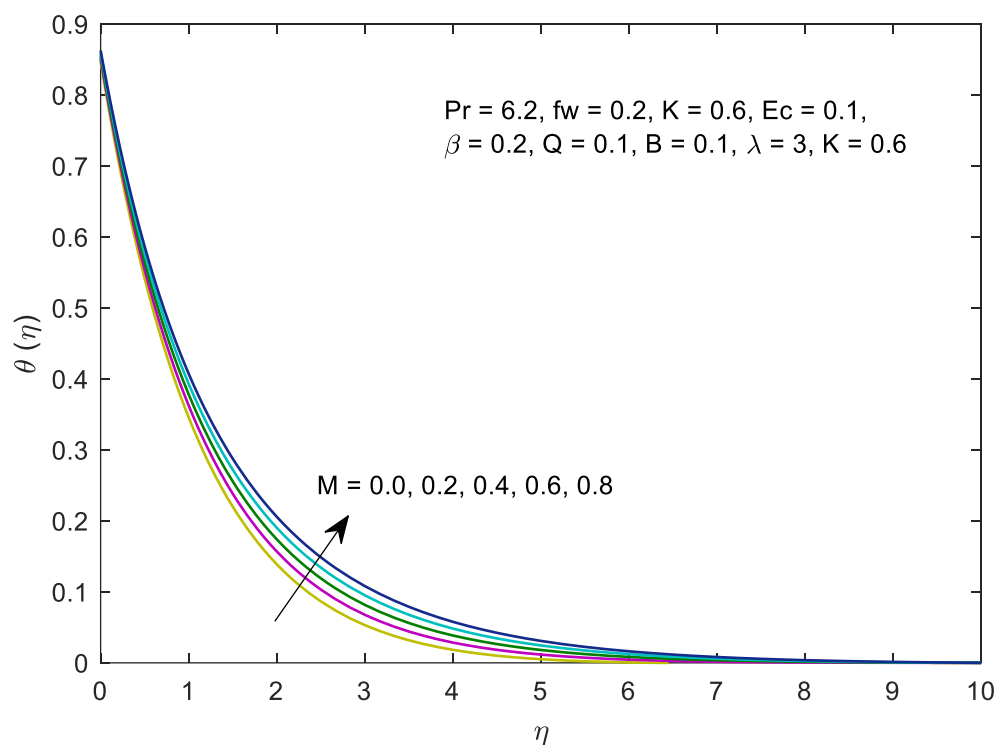


Fig.2c Temperature for different values of  $M$

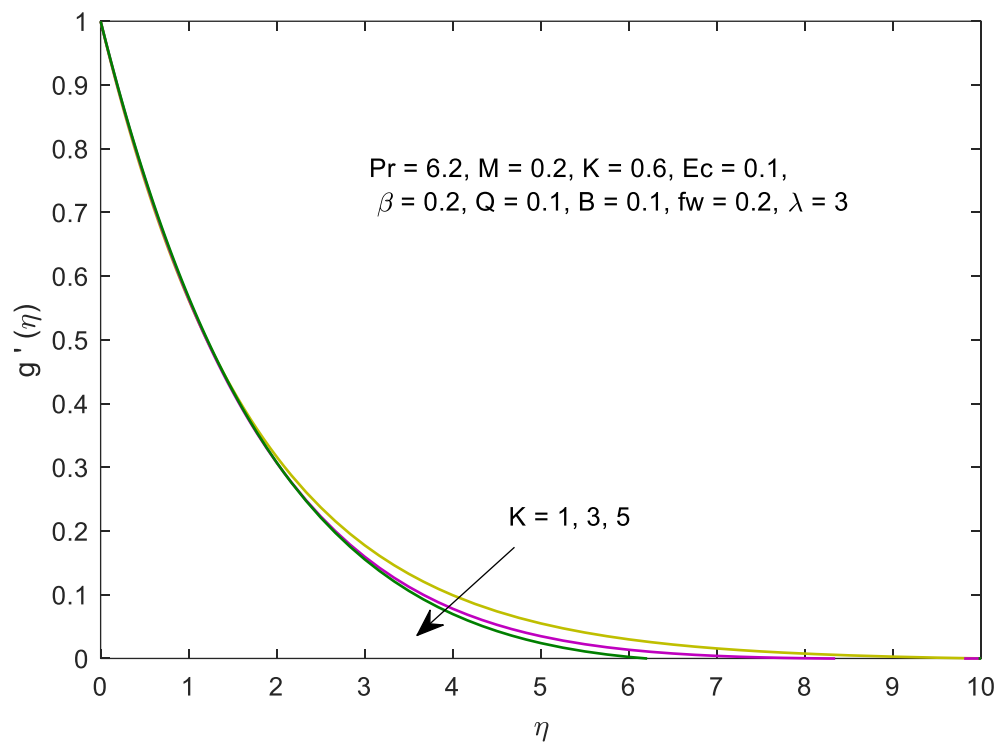


Fig.3a Velocity for different values of  $K$

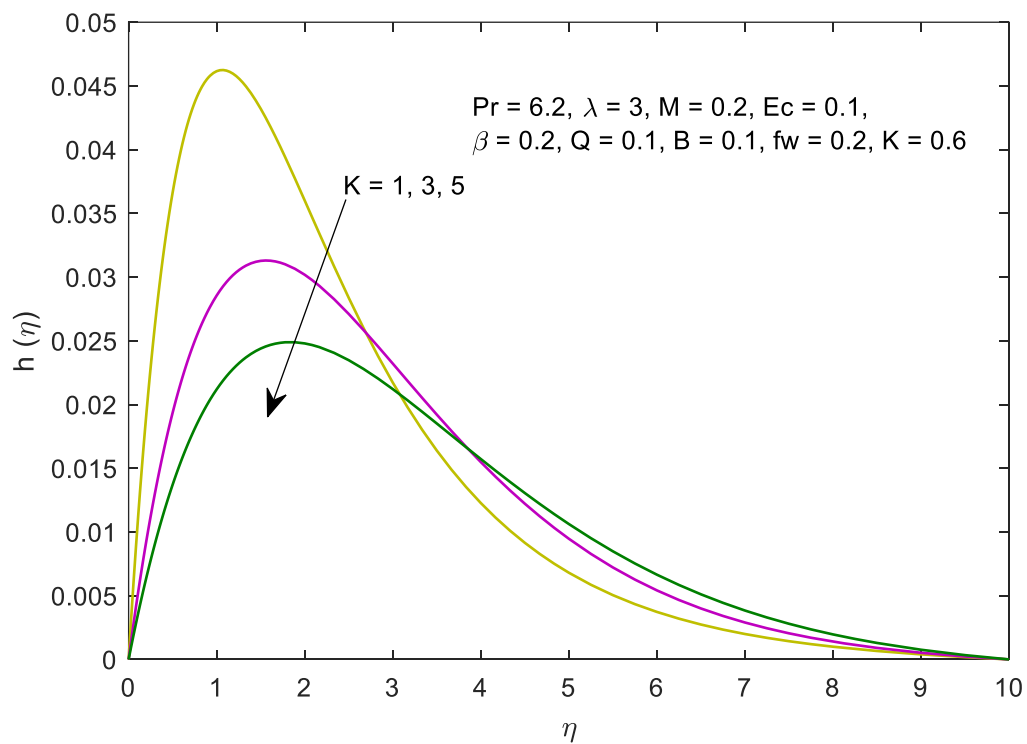


Fig.3b Angular velocity for different values of  $K$

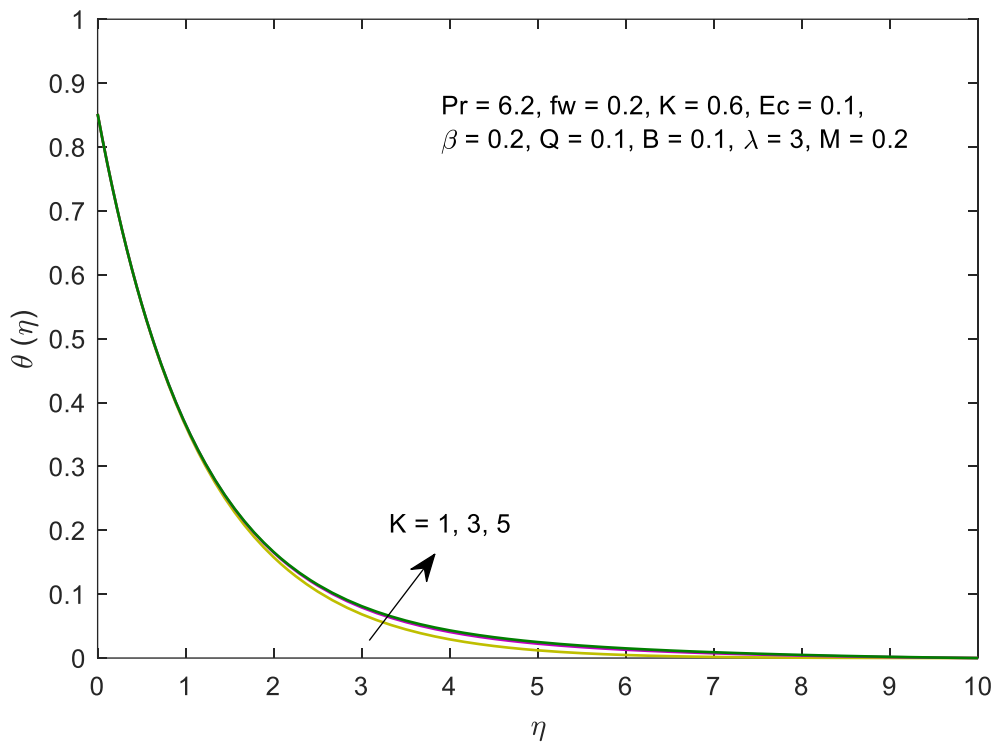


Fig.3c Temperature for different values of  $K$

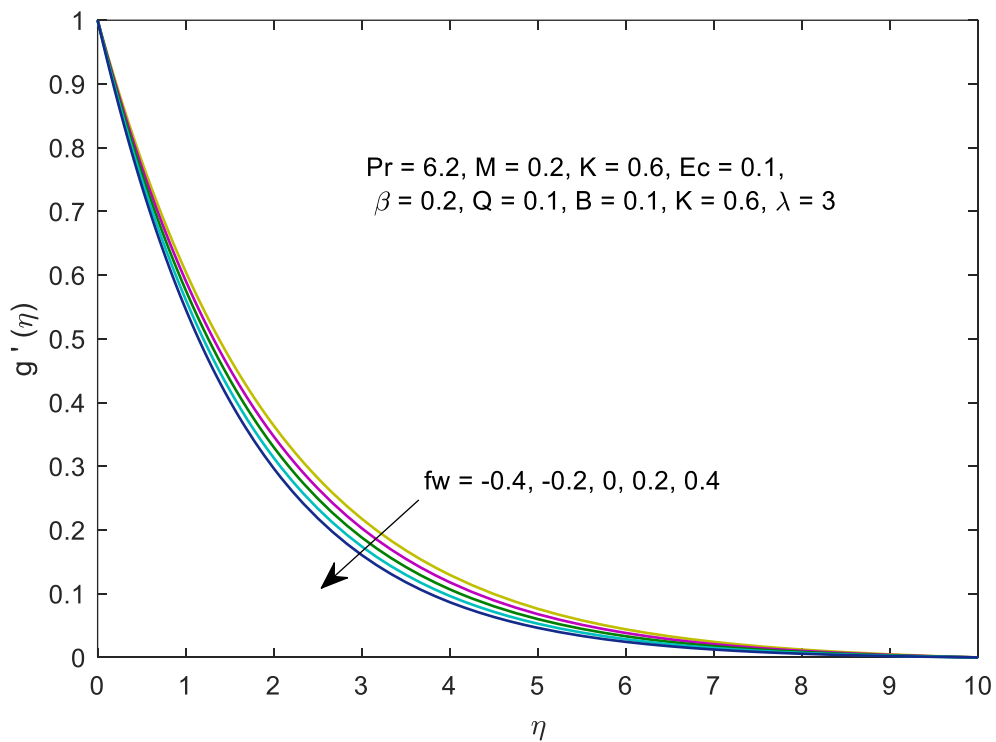


Fig.4a Velocity for different values of  $f_w$

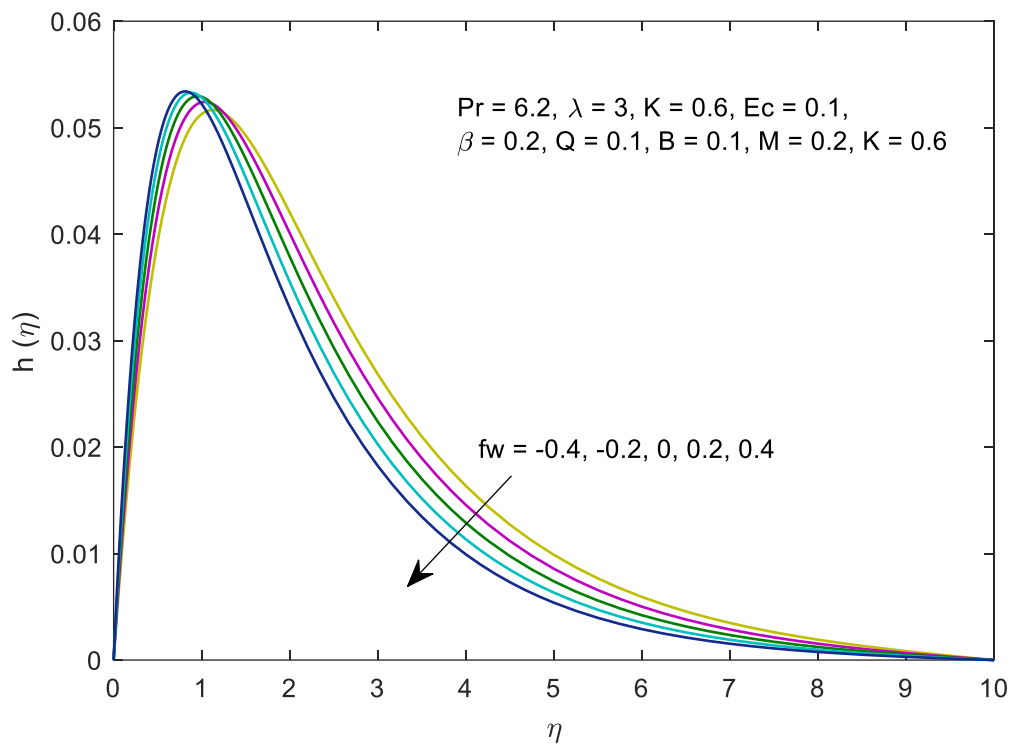


Fig.4b Angular velocity for different values of  $fw$

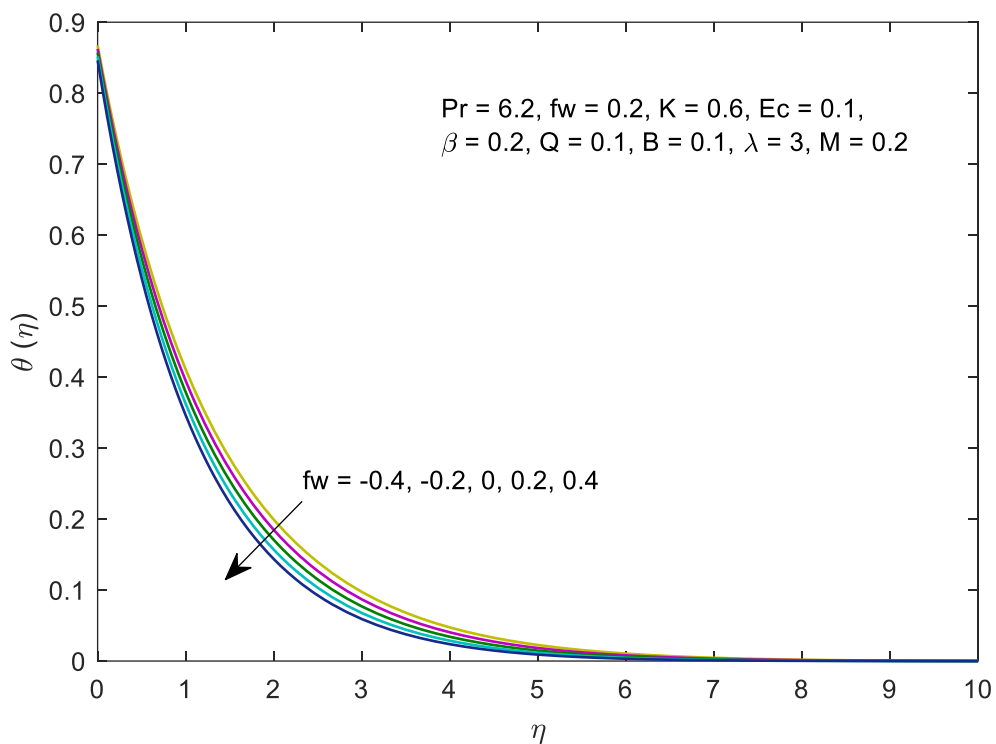


Fig.4c Temperature for different values of  $fw$

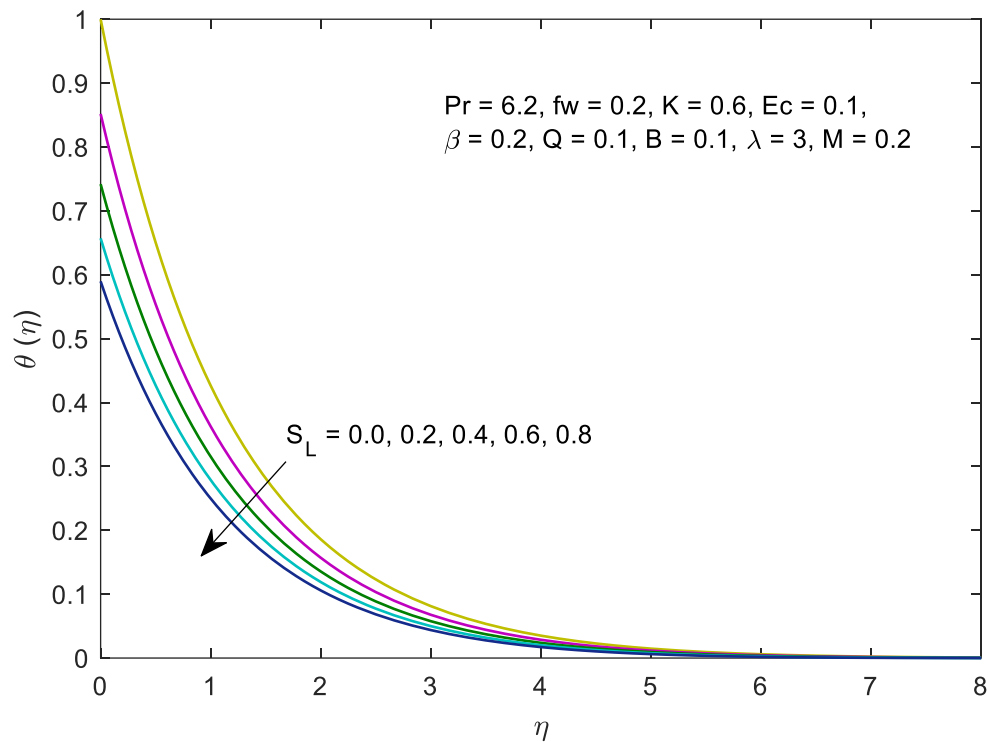


Fig.5 Temperature for different values of  $\beta$

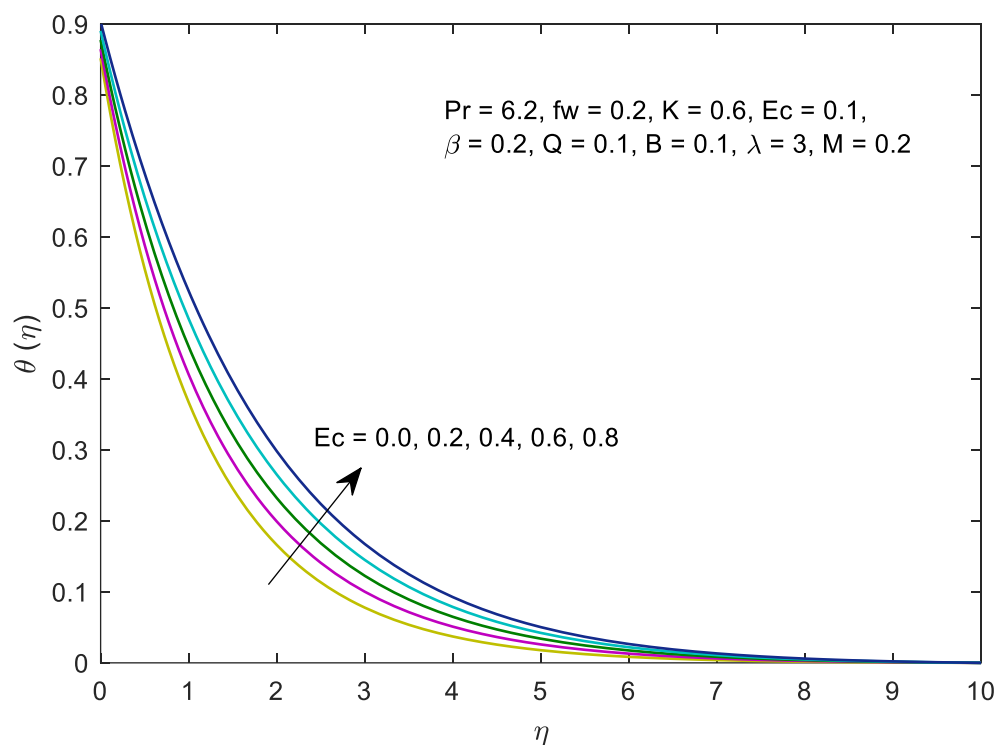


Fig.6 Temperature for different values of  $Ec$ .

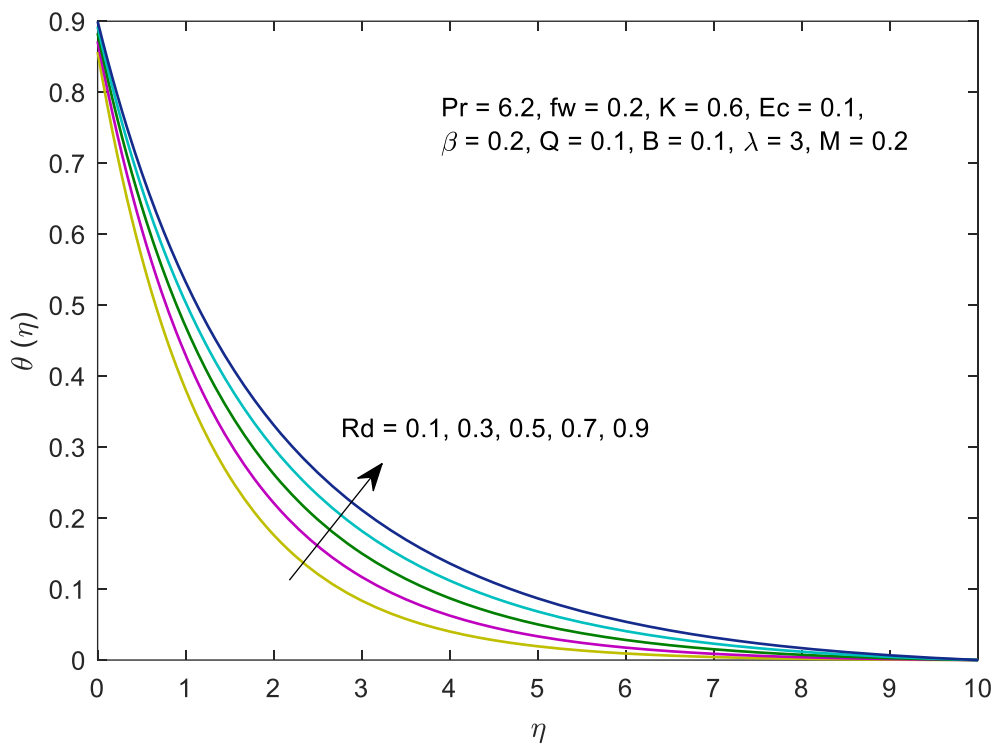


Fig.7 Temperature for different values of  $R$ .

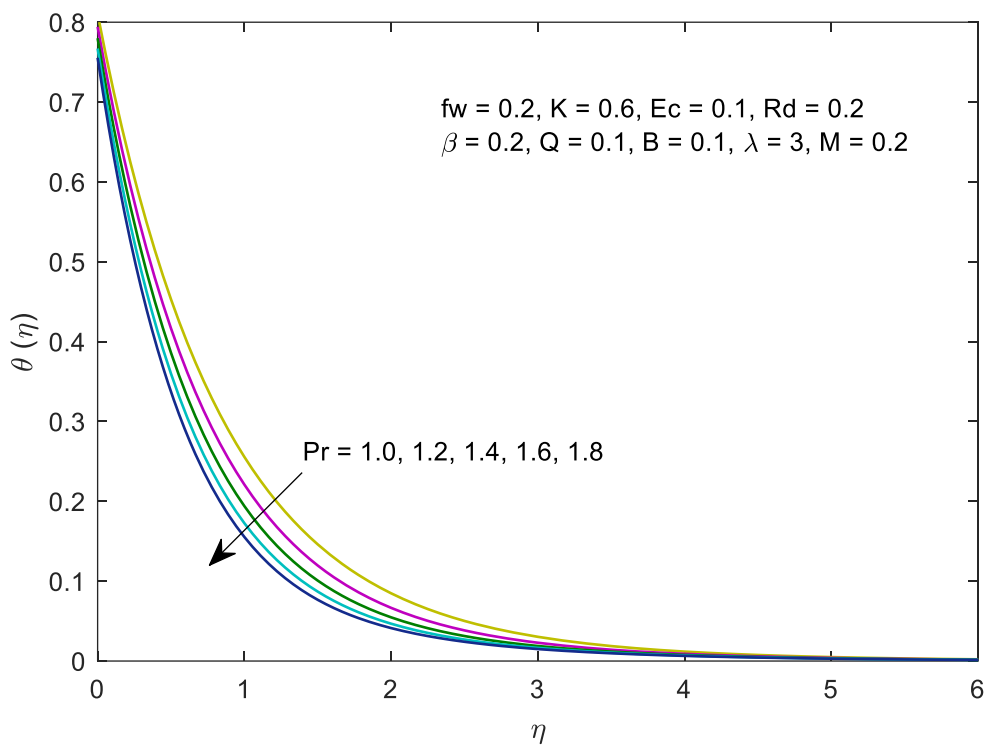


Fig.8 Temperature for different values of  $Pr$ .

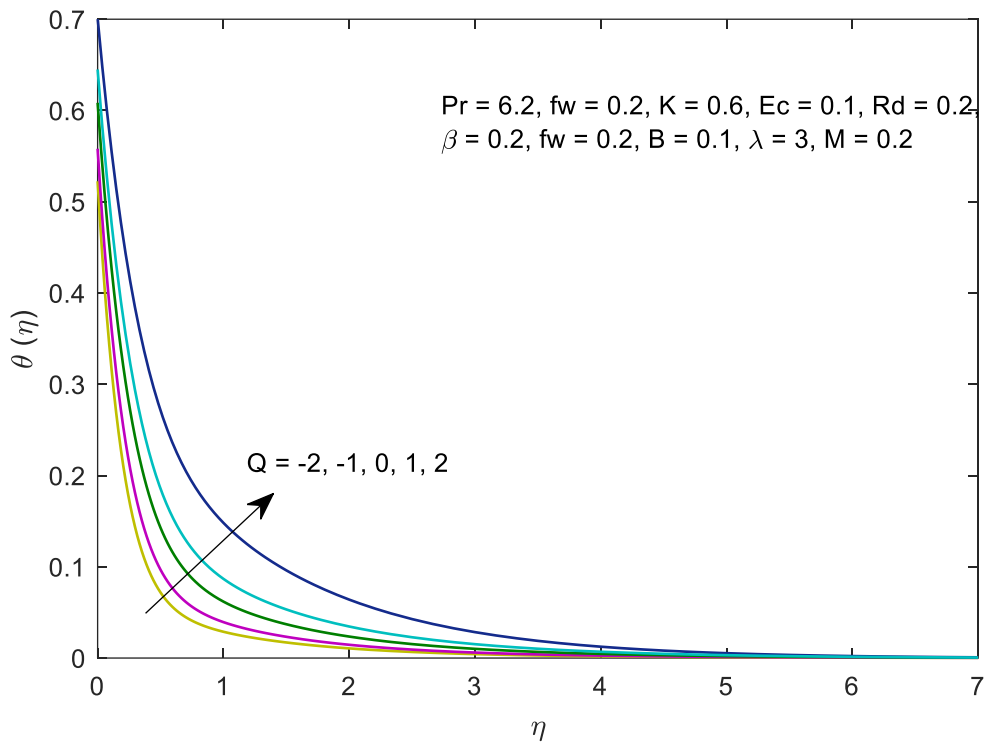


Fig.9 Temperature for different values of  $Q$ .

Table 1 Numerical values for  $-g''(0)$ ,  $-h'(0)$ ,  $-\theta''(0)$  for different parametric values.

$\lambda$	$M$	$K$	$fw$	$-g''(0)$	$-h'(0)$	$-\theta''(0)$
0.5	0.2	0.6	0.2	0.960571	-0.045146	1.922588
1.5				0.729615	-0.105967	1.925899
2.5				0.610596	-0.145859	1.925968
3.0	0.0			0.522305	-0.151613	2.002755
	0.5			0.633352	-0.173470	1.813333
	1.0			0.727487	-0.190283	1.633955
	0.2	0.5		0.569860	-0.182144	1.925547
		1.0		0.567410	-0.113750	1.926122
		1.5		0.565468	-0.085708	1.926778
		0.6	-0.4	0.495901	-0.116098	1.439815
			0.0	0.543667	-0.145386	1.760429
			0.4	0.596125	-0.177325	2.087133

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