



K₀,K₁,K₂,K₃,K₄,K₅, K₆ CONSTANTS EVALUATION OF MAGNETO-CRYSTALLINE ANISOTROPY ENERGY DENSITY EQUATION OF ELECTRICAL STEEL BASED ON TEXTURE FACTOR FOR α , α^* , η ,RANDOM IDEAL FIBRES

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1. ABSTRACT:

Texture Factor, A* and Magnetic Crystalline Anisotropy Energy Density K₀,K₁,K₂,K₃,K₄,K₅, K₆ Constants are important parameters for Electrical Steels. While the former indicates volume density of crystals having preferred Orientation, latter indicates the easy and hard magnetization directions. Evaluation of these parameters for Pure Iron and Electrical Steel enables in reduction of core losses and improving the electrical energy efficiency in Transformers, Rotating Machines. In this research article, an attempt is made to compute Magneto-Crystalline Anisotropy Energy Density for pure iron based on Texture Factor for Ideal fibers.

Keywords: Texture Factor, Magnetic Crystalline Anisotropy Energy Density, Core losses

2. INTRODUCTION:

The Magneto Crystalline Anisotropy constants K₀,K₁,K₂,K₃,K₄,K₅, K₆ values determine the extent to which a material is easily magnetizable. Their value depends on Chemical Composition, Crystal Structure, and Thermo-Mechanical Processing history of the given material. In material science, Texture Factor is an important microstructural parameter which directly determines the anisotropy degree of most physical properties of a polycrystalline material at the macro scale. Its characterization is thus of fundamental and applied importance, and

should ideally be performed prior to any physical property measurement or modeling. Neutron diffraction is a tool of choice for characterizing crystallographic texture. The obtained information is representative of a large number of grains, leading to a better accuracy of the statistical description of texture. Texture factor constants K₀,K₁,K₂,K₃,K₄,K₅, K₆ values determines the preferred orientations of grains, the Overall Texture Factor is quantitative measurement of texture. The value signifies extent of presence of standard texture viz. η Texture (T.F = 27.88), α Texture (T.F = 30.06, α^* Texture (T.F = 30.77) ,Random Texture(T.F = 31.88) in the given material.

3.STANDARD EQUATIONS:

$$E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2)^2 ;$$

$$A^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2)^2 ;$$

$$E^* = 0.355A^* + (0.163 - 0.031A^*)[\text{wt\%Si}] - 1.898$$

3. ESTIMATION OF MAGNETIC ANISOTROPY CONSTANTS K₀,K₁,K₂,K₃,K₄,K₅, K₆ CONSTANTS EVALUATION OF FOR ELECTRICAL STEELS:

Magneto Crystalline Anisotropy Energy is generally expressed by an expansion into direction cosines $\alpha_1, \alpha_2, \alpha_3$ of the magnetization with respect to the crystal axes.

$$E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2) [I] ;$$

$$E^* = 0.355A^* + (0.163 - 0.031A^*)[\text{wt\%Si}] - 1.898$$

Texture Factor, A* for η fibre is $\langle 001 \rangle$ // RD is 27.88

Texture Factor, A* for α fibre is $\langle 110 \rangle$ // RD is 30.06

Texture Factor, A* for α^* fibre is $\langle 112 \rangle$ // RD is 30.77

Texture Factor, A* for random fibre is 31.88

E* for η fibre is $\langle 001 \rangle$ // RD is $\Rightarrow E^* = 0.355 * 27.88 - 1.898 - 0.70128 [\text{wt}\% \text{Si}] = 7.9994 - 0.70128 [\text{wt}\% \text{Si}]$

E* for α fibre is $\langle 110 \rangle$ // RD is $\Rightarrow E^* = 0.355 * 30.06 - 1.898 - 0.76886 [\text{wt}\% \text{Si}] = 8.7733 - 0.76886 [\text{wt}\% \text{Si}]$

E* for α^* fibre is $\langle 112 \rangle$ // RD is $\Rightarrow E^* = 0.355 * 30.77 - 1.898 - 0.79087 [\text{wt}\% \text{Si}] = 9.02535 - 0.79087 [\text{wt}\% \text{Si}]$

E* for random texture is $\Rightarrow E^* = 0.355 * 31.88 - 1.898 - 0.82528 [\text{wt}\% \text{Si}] = 9.4194 - 0.82528 [\text{wt}\% \text{Si}]$

$E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2)^2 [I];$

FOR $[100]$ directions, $\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0$

$E^*_{[100]} = K_0 = 7.9994 [II]$

$E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2)^2 [I];$

FOR $[110]$ directions, $\alpha_1 = 1/\sqrt{2}, \alpha_2 = 1/\sqrt{2}, \alpha_3 = 0$

$8.7733 - 0.76886 [\text{wt}\% \text{Si}] = 7.9994 - 0.70128 [\text{wt}\% \text{Si}] + K_1/4 + K_3/16 + K_5/64$

$16 K_1 + 4 K_3 + K_5 = [8.7733 - 7.9994] * 64 - [0.06758 [\text{wt}\% \text{Si}]] * 64 = 0.7739 * 64 - 4.32512 [\text{wt}\% \text{Si}] = 49.5296 - 4.32512 [\text{wt}\% \text{Si}]$

$16 * 3 + 4 * 0.3 + 0.3296 - 4.32512 [\text{wt}\% \text{Si}] = 49.5296 - 4.32512 [\text{wt}\% \text{Si}]$

$K_1 = 3; K_3 = 0.3; K_5 = 0.3296 - 4.32512 [\text{wt}\% \text{Si}] \dots [III]$

$E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2)^2 [I];$

For random, $E^* = 9.4194 - 0.82528 [\text{wt}\% \text{Si}] \Rightarrow \alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 1$ ARE ASSUMED

$9.4194 - 0.82528 [\text{wt}\% \text{Si}] = 7.9994 - 0.70128 [\text{wt}\% \text{Si}] + 3 K_1 + K_2 + 9 K_3 + 3 K_4 + 27 K_5 + K_6$

$1.42 - 0.124 [\text{wt}\% \text{Si}] = 3(K_1 + 3K_3 + 9K_5) + 3K_4 + K_2 + K_6$

$K_1 = 3; K_3 = 0.3; K_5 = 0.3296 - 4.32512 [\text{wt}\% \text{Si}] \dots \text{FROM } [III]$

$3K_4 + K_2 + K_6 = -19.1792 + 116.65424 [\text{wt}\% \text{Si}];$

Dividing by 3, we have

$K_4 + 0.333K_2 + 0.333K_6 = -6.39306 + 38.884 [\text{wt}\% \text{Si}]; \dots [IV]$

E* for α^* fibre is $\langle 112 \rangle$ // RD is $\Rightarrow A^* = 30.77; E^* = 9.02535 - 0.79087 [\text{wt}\% \text{Si}]$

FOR $[112]$ directions, $\alpha_1 = 0.408248, \alpha_2 = 0.408248, \alpha_3 = 0.816496;$

$\text{COS} \langle 112 \rangle, \langle 100 \rangle = \alpha_1 = (1.1 + 1.0 + 2.0) / (\sqrt{6} * \sqrt{1}) = 0.408248$

$\text{COS} \langle 112 \rangle, \langle 010 \rangle = \alpha_2 = (0.1 + 1.1 + 2.0) / (\sqrt{6} * \sqrt{1}) = 0.408248$

$\text{COS} \langle 112 \rangle, \langle 001 \rangle = \alpha_3 = (0.1 + 1.0 + 2.1) / (\sqrt{6} * \sqrt{1}) = 0.816496$

$(\sum \alpha_1^2 \alpha_2^2) = 0.24999928 \approx 0.25; (\prod \alpha_1^2) = 0.0185184$

$E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2)^2$

$9.02535 - 0.79087 [\text{wt}\% \text{Si}] = 7.9994 - 0.70128 [\text{wt}\% \text{Si}] + K_1 (0.25) + K_2 (0.0185184) + K_3 (0.25)^2 + K_4 (0.25) (0.0185184) + K_5 (0.25)^3 + K_6 (0.0185184)^2$

Dividing $K_1 (0.25) + K_2 (0.0185184) + K_3 (0.25)^2$ by $(0.25)^2$

Dividing $K_4 (0.25) (0.0185184) + K_5 (0.25)^3 + K_6 (0.0185184)^2$ by $(0.25)^3$

On Simplification, we have

$[K_1 + 0.25 K_3 + K_5] + [0.07407378 K_2 + 0.296296 K_4 + 0.02194778 K_6] = 0.01602095 - 0.08959 [\text{wt}\% \text{Si}] * [0.015624865]$

$[K_1 + 0.25 K_3 + K_5] + [0.07407378 K_2 + 0.296296 K_4 + 0.02194778 K_6] = 0.01602095 - 0.00139983 [\text{wt}\% \text{Si}]$

$K_1 = 3; K_3 = 0.3; K_5 = 0.3296 - 4.32512 [\text{wt}\% \text{Si}] \dots \text{FROM } [III]$

$[0.07407378 K_2 + 0.296296 K_4 + 0.02194778 K_6] = 0.01602095 - 3 * -0.25 * 0.3 - 0.3296 + 4.32512 [\text{wt}\% \text{Si}] - 0.00139983 [\text{wt}\% \text{Si}]$

$[0.07407378 K_2 + 0.296296 K_4 + 0.02194778 K_6] = -3.388579 - 4.32372017 [\text{wt}\% \text{Si}] \dots [V]$

Dividing by, 0.296296 on both sides

...[IV] (-) ...[V] ; we have

$$K_4 + 0.333K_2 + 0.333K_6 = -6.39306 + 38.884[\text{wt}\% \text{Si}]$$

$$K_4 + 0.25K_2 + 0.07407378 K_6 = -11.436465 - 14.592570[\text{wt}\% \text{Si}]$$

$$0.083 K_2 + 0.2589262 K_6 = 5.043405 + 53.476570[\text{wt}\% \text{Si}]$$

Dividing by, 0.2589262 on both sides, we have

$$0.32055 K_2 + K_6 = 19.478156 + 206.5320[\text{wt}\% \text{Si}] = 14 + 206.5320[\text{wt}\% \text{Si}] + (5.478156);$$

$$\text{ASSUMING, } K_6 = 14 + 206.5320[\text{wt}\% \text{Si}] ; K_2 = (5.478156)/(0.32055) = 17.0898;$$

$$\text{We have } 3K_4 + K_2 + K_6 = -19.1792 + 116.65424 [\text{wt}\% \text{Si}];;$$

$$3K_4 + 17.0898 + 14 + 206.5320[\text{wt}\% \text{Si}] = -19.1792 + 116.65424 [\text{wt}\% \text{Si}];;$$

$$K_4 = -16.7563 - 29.959253[\text{wt}\% \text{Si}];$$

$$\text{Finally, we have } K_0 = 7.9994 - 0.70128 [\text{wt}\% \text{Si}]; K_1 = 3; K_2 = 17.0898; K_3 = 0.3 ; K_4 = -16.7563 - 29.959253[\text{wt}\% \text{Si}]; K_5 = 0.3296 - 4.32512[\text{wt}\% \text{Si}] ; K_6 = 14 + 206.5320[\text{wt}\% \text{Si}]$$

$$\text{We have, } E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2)^2$$

$$\text{We have, } E^* = 7.9994 - 0.70128[\text{wt}\% \text{Si}] + 3(\sum \alpha_1^2 \alpha_2^2) + 17.0898(\prod \alpha_1^2) + 0.3(\sum \alpha_1^2 \alpha_2^2)^2 - [16.7563 + 29.959253[\text{wt}\% \text{Si}](\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + [0.3296 - 4.32512[\text{wt}\% \text{Si}](\sum \alpha_1^2 \alpha_2^2)^3 + [14 + 206.5320[\text{wt}\% \text{Si}](\prod \alpha_1^2)^2 \dots [VI]$$

$$E^* = E^*_{\text{IRON}} - E^{**}$$

$$\text{Where } E^*_{\text{IRON}} = 7.9994 + 3(\sum \alpha_1^2 \alpha_2^2) + 17.0898(\prod \alpha_1^2) + 0.3(\sum \alpha_1^2 \alpha_2^2)^2 - 16.7563(\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + 0.3296(\sum \alpha_1^2 \alpha_2^2)^3 + 14(\prod \alpha_1^2)^2$$

$$E^{**} = -0.70128[\text{wt}\% \text{Si}] + 29.959253[\text{wt}\% \text{Si}](\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) - 4.32512[\text{wt}\% \text{Si}](\sum \alpha_1^2 \alpha_2^2)^3 + 206.5320[\text{wt}\% \text{Si}](\prod \alpha_1^2)^2$$

ABOVE EQUATION IS THE STANDARD EQUATION FOR COMPUTATION OF MAGNETIC-CRYSTALLINE ANISOTROPIC ENERGY DENSITY FOR ELECTRICAL STEELS.

CRYSTALLOGRAPHIC DIRECTION	MAGNETO-CRYSTALLINE ANISOTROPY ENERGY DENSITY, E^*	MAGNETO-CRYSTALLINE ANISOTROPY ENERGY DENSITY, $E^{*0.51\%Si}$	MAGNETO-CRYSTALLINE ANISOTROPY ENERGY DENSITY, $E^{*1.38\%Si}$	MAGNETO-CRYSTALLINE ANISOTROPY ENERGY DENSITY, $E^{*2.8\%Si}$	MAGNETO-CRYSTALLINE ANISOTROPY ENERGY DENSITY, $E^{*3.2\%Si}$
[100] $\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0$	7.9994 - 0.70128[wt %Si]	7.6417472	7.0316336	6.035816	5.755304
[110] $\alpha_1 = 1/\sqrt{2}, \alpha_2 = 1/\sqrt{2}, \alpha_3 = 0$	8.7733 - 0.76886[wt %Si]	8.3811814	7.7122732	6.620492	6.312948
[112] $\alpha_1 = 0.408248, \alpha_2 = 0.408248, \alpha_3 = 0.816496$ ($\sum \alpha_i^2 = 1$) ($\prod \alpha_i^2 = 0.0185184$)	9.02535 - 0.79087[wt %Si]	8.6220063	7.9339494	6.810914	6.494566
Random, $\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 1$	9.4194 - 0.82528[wt %Si]	8.9985072	8.2805136	7.108616	6.778504

DISCUSSION:

The <100>//ND fibre accounts for the lowest anisotropy energy since the flux lines, distributed homogenously in a plane of the rotating laminated sheet, have an easiest magnetization direction with the in-plane rotated cube texture components. On the contrary, the <112> and the <110>//RD, Random fibre orientations have relatively high anisotropy energy and as such, the occurrence of these components in pure iron is undesirable.

4. ESTIMATION OF TEXTURE FACTOR $K_0, K_1, K_2, K_3, K_4, K_5, K_6$ CONSTANTS FOR PURE IRON

$$E^* = 7.9994 - 0.70128[\text{wt}\%Si] + 3(\sum \alpha_i^2 \alpha_j^2) + 17.0898(\prod \alpha_i^2) + 0.3(\sum \alpha_i^2 \alpha_j^2)^2 - [16.7563 + 29.959253[\text{wt}\%Si](\sum \alpha_i^2 \alpha_j^2)(\prod \alpha_i^2) + [0.3296 - 4.32512[\text{wt}\%Si](\sum \alpha_i^2 \alpha_j^2)^3 + [14 + 206.5320[\text{wt}\%Si](\prod \alpha_i^2)^2] \dots [VI]$$

From Standard Equation $E^* = 0.355A^* + (0.163 - 0.031A^*)[\text{wt}\%Si] - 1.898$;

$$[0.355 - 0.031[\text{wt}\%Si]] A^* = 7.9994 + 1.898 - 0.163[\text{wt}\%Si] + 3(\sum \alpha_i^2 \alpha_j^2) + 17.0898(\prod \alpha_i^2) + 0.3(\sum \alpha_i^2 \alpha_j^2)^2 - [16.7563 + 29.959253[\text{wt}\%Si](\sum \alpha_i^2 \alpha_j^2)(\prod \alpha_i^2) + [0.3296 - 4.32512[\text{wt}\%Si](\sum \alpha_i^2 \alpha_j^2)^3$$

$$+ [14 + 206.5320[\text{wt}\% \text{Si}]] (\sum \alpha_i^2)^2$$

We have,

$$K_0 = \frac{[9.8974 - 0.163[\text{wt}\% \text{Si}]]}{[0.355 - 0.031[\text{wt}\% \text{Si}]]} ; K_1 = \frac{3}{[0.355 - 0.031[\text{wt}\% \text{Si}]]} ; K_2 = \frac{17.0898}{[0.355 - 0.031[\text{wt}\% \text{Si}]]}$$

$$K_3 = \frac{0.3}{[0.355 - 0.031[\text{wt}\% \text{Si}]]} ; K_4 = \frac{-[16.7563 + 29.959253[\text{wt}\% \text{Si}]]}{[0.355 - 0.031[\text{wt}\% \text{Si}]]} ; K_5 = \frac{[0.3296 - 4.32512[\text{wt}\% \text{Si}]]}{[0.355 - 0.031[\text{wt}\% \text{Si}]]}$$

$$K_6 = \frac{[14 + 206.5320[\text{wt}\% \text{Si}]]}{[0.355 - 0.031[\text{wt}\% \text{Si}]]}$$

ABOVE EQUATION IS THE TEXTURE FACTOR EQUATION FOR ELECTRICAL STEELS WITH SEVEN CONSTANTS.

FOR <100> direction , , $\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0, A^* = 27.88 \Rightarrow A^* = 27.88$ for η fibre <100>//RD

FOR <110> direction , $\alpha_1 = 1/\sqrt{2}, \alpha_2 = 1/\sqrt{2}, \alpha_3 = 0, \Rightarrow A^* = 30.06$ for α fibre <110>//RD

FOR <112> direction , $\alpha_1 = 0.408248, \alpha_2 = 0.408248, \alpha_3 = 0.816496, A^* = 30.77$ for α^* fibre <110>//RD

FOR random direction(assumed), $\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 1, A^* = 31.88 \Rightarrow A^* = 31.88$ for random fibre

5. CONCLUSIONS:

Magneto-Crystalline Anisotropy Energy Density value is least for [100] directions, and higher for [110] & [111] directions. Therefore [100] directions are easy directions of magnetization for pure iron and [112] hardest direction for magnetization of pure iron, [110] direction is harder direction for magnetization of pure iron. Texture Factor Equation results are consistent with the standard results and conforms to the value of ideal fibres.

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